Sampling Theory and Computing Eigenvalues of Discontinuous Dirac Systems by Using Regularized Sinc Method

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Abstract

In this paper we apply a regularized sinc method to compute the eigenvalues of a discontinuous Dirac systems, which contain eigenvalue parameter in one boundary condition, with transmission conditions at the point of discontinuity. The regularized technique allows us to insert some parameters to the well known sinc method; strengthening the existing technique and to avoid the aliasing error. The error analysis is established considering both truncation and amplitude errors associated with the sampling theorem. Numerical examples with tables and illustrative figures are given.

Keywords: Sinc methods; Dirac systems; transmission conditions; discontinuous boundary value problems; truncation and amplitude errors.

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