An inverse fractional abstract Cauchy problem with nonlocal conditions

Mahmoud M. El-Borai , Khairia El-Said El-Nadi

m_m_elborai@yahoo.com, khairia_el_said@hotmail.com

Faculty of Science, Alexandria University, Alexandria, Egypt

Abstract

This note is devoted to the study of an inverse Cauchy problem in a Hilbert space H for the abstract fractional differential equation of the form:

$$\frac{d^{\alpha} u(t)}{dt^{\alpha}} \;=\; A\; u(t)\; +\; f(t)\; g(t),$$

with the nonlocal initial condition:

$$u(0) = u_0 + \sum_{k=1}^{p} c_k u(t_k),$$

and the overdetermination condition:

$$(u(t), v) = w(t),$$

where $(.,.)$ is the inner product in H, f is a real unknown function w is a given real function, u_0, v are given elements in H, g is a given abstract function with values in H, $0 < \alpha \leq 1$, u is unknown, and A is a linear closed operator defined on a dense subset of H.

It is supposed that A generates a bounded semigroup. An application is given to study an inverse problem in a suitable Sobolev space for general fractional parabolic partial differential equations with unknown source functions.

Keywords and phrases: Fractional abstract differential equations, nonlocal initial conditions, inverse Cauchy problem.

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