An inverse problem for the wave equation with one measurement

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We consider the wave equation $(\partial_t^2 - \Delta_g)u(t,x) = f(t,x)$, in $\mathbb{R}^n$, $u|_{\mathbb{R}_- \times \mathbb{R}^n} = 0$, where the metric $g = (g_{jk}(x))_{j,k=1}^n$ is known outside a compact set $M \subset \mathbb{R}^n$. The source term $f(t,x)$ is deterministic sum of point sources, $f(t,x) = \sum_{j=1}^\infty a_j \delta_{x_j}(x)\delta(t)$ where the points $x_j,\ j \in \mathbb{Z}_+$ form dense set on ∂M. We call f the pseudorandom noise. It is shown that when a_j are chosen appropriately, then $u|_{\mathbb{R}\times\partial M}$ determines the scattering relation on ∂M. In particular, the distances $d_M(x,y)$ for all boundary points $x, y \in \partial M$ are determined. In the case when $(M, g|_M)$ is a simple Riemannian manifold and g is conformally Euclidian, these distance are known to determine the metric g in M. Finally, we discuss the prospects of applying these ideas to random sources.