Useful Hypothesis in Inverse Problems of Interpretation

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In this paper one possible approach to the statement of interpretation inverse problems and the justification of practical application of approximate solutions are proposed [1,2].

Let's present an inverse problem of interpretation as solution of equation

$$\tilde{A}z = u_\delta , \tag{1}$$

where $\tilde{A}$ is compact operator, u_δ is initial data, $z \in Z$, $u_\delta \in U$ (Z, U are functional spaces with norm).

Further we shall believe that error of function u_δ from the exact function u_{ex} has the given size δ :

$$\| u_\delta - u_{ex} \|_X \leq \delta . \tag{2}$$

In inverse problems of interpretation it is necessary to take into account the inaccuracy of operator $\tilde{A}$ in relation to the exact operator A_{ex} additionally [1,3].

Let us suppose that the characteristic of an error of the operator $\tilde{A}$ is given:

$$\| \tilde{A} - A_{ex} \|_{Z \to U} \leq h . \tag{3}$$

The set of possible solution of equation (1) is necessary to extend to set $Q_{\delta,h}$ taking into account the inaccuracy of the operator $\tilde{A}$:

$$Q_{\delta,h} = \{ z : z \in Z, \| \tilde{A}z - u_\delta \|_U \leq h \| z \|_Z + \delta \} .$$

The algorithm for the solution of the incorrect problem with approximate operator was proposed in work [4] which is based on Tikhonov's regularization method [5].

The statement of such interpretation inverse problem can be formulated as follows for obtaining of the stable solution: it is necessary to find an element $z_{est} \in Q_{\delta,h}$ on which the greatest lower bound of some stabilizing functional $\Omega[z]$ is reached

$$\inf_{z \in Q_{\delta,h} \cap Z_1} \Omega[z] = \Omega[z_{est}], \qquad (4)$$

where Z_1 is subset of Z, on subset Z_1 has been defined stabilizing functional $\Omega[z]$, the set Z_1 is everywhere dense in Z [5].

One of the important characteristics for the specified algorithm is the size h of an error. The definition of h represents significant difficulties, as the exact operator A_{ex} is unknown.

By result of the solution of interpretation inverse problem it is necessary to accept some approximation $\tilde{z}$ to the exact solution z_{ex} of the equation (1) or its estimation z_{est} in the beforehand certain sense [1,2].

The functional $\Omega[z]$ can characterize the chosen property of the exact solution (for example, smoothness). The approximated solution will give the estimation from below of exact solution on a degree of smoothness. If a functional $\Omega[z]$ characterizes a deviation of the approximate solution from the given function z_{ap}, then the solution of an extreme problem (4) will give function from set $Q_{\delta,h} \cap Z_1$ closest to function z_{ap}. Thus it is obvious that z_{ap} should not belong to set $Q_{\delta,h} \cap Z_1$.

The estimation of a deviation of the operator $\tilde{A}$ from exact operator A_{ex} cannot be made essentially at a consideration of interpretation problems.

For overcoming the specified difficulties it is offered to accept the following ***hypothesis***: for the exact solution z_{ex} of the equation $A_{ex}\, z = u_{ex}$ the inequality is valid

$$\Omega[z_{ex}] \geq \Omega[\tilde{z}], \qquad (4)$$

where $\tilde{z}$ is regularized solution of equation (1) with approximate operator $\tilde{A}$ and approximate initial data u_δ, $\Omega[z]$ is stabilized functional [5].

The offered ***hypothesis*** don't use the size of inaccuracy h of the operator $\tilde{A}$ from the exact operator A_{ex}, which cannot be defined essentially, at the solution of inverse problems of interpretation.

The satisfaction of an inequality (4) is obvious if the operators A_{ex}, $\tilde{A}$ are linear. For the nonlinear operator A_{ex} (that in the greater degree corresponds to a reality) the inequality (4) can be proved by properties of the approximated operators which are used in calculations [6].

Use of the offered hypothesis allows to receive various objective estimations of the exact solution z_{ex} of inverse problems such as (1) that is important in recognition problems [1,2]. Moreover the size h is not used in calculations. At $\tilde{A} \to A_{ex}$ the estimation of function z_{ex} will be more exact. For definition of parameter regularization it is possible to use a usual discrepancy method [5] where the value h is absent.

Offered algorithm can be use also for estimation of real unknown parameters of physical process by identification method.

References

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