Acceleration of the modified alternating iterative algorithm by the conjugate gradient method for the Cauchy problem for the Helmholtz equation

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Let Ω be a bounded domain in $\mathbb{R}^n$ with a Lipschitz boundary Γ divided into two parts Γ_0 and Γ_1 which do not intersect one another and have a common Lipschitz boundary. We consider the following Cauchy problem for the Helmholtz equation

$$\begin{cases} \Delta u + k^2 u = 0 & \text{in} \quad \Omega, \\ u = f & \text{on} \quad \Gamma_0, \\ \partial_\nu u = g & \text{on} \quad \Gamma_0, \end{cases}$$

where k is the wave number, ∂_ν denotes the outward normal derivative, and f and g are specified Cauchy data on Γ_0. This problem is ill–posed.

In [3], we developed a modification of the alterating iterative algorithm that we used to solve this problem. This modification is based on the alternating iterative schemes suggested in [1] and [2] since the latter diverge for the large constant k^2 in the Helmholtz equation. The numerical experiments confirmed that the proposed modification works well but it requires a big numer of iteration. In the current work, we present an acceleration of the modified alternating algorithm using the conjugate gradient method and present some numerical results.

References

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