Cordial Volterra integral equations of the first kind

Gennadi Vainikko (gennadi.vainikko@ut.ee)

University of Tartu, J.Liivi 2, 50409 Tartu, Estonia

We study the first kind cordial [1,2,5,6] Volterra integral equation

$$\int_0^t \frac{1}{t}\varphi\Big(\frac{s}{t}\Big)a(t,s)u(s)ds = f(t),\ 0 < t \le T,\ \text{ or }\ V_{\varphi,a}u = f. \tag{1}$$

We assume that $\varphi : (0,1) \to \mathbb{R}$ and $a : \triangle_T = \big\{(t,s) : 0 \le s \le t \le T\big\} \to \mathbb{R}$ satisfy

$$\int_0^1 x^r \mid \varphi(x) \mid dx < \infty,\ \int_0^1 x^r\varphi(x)dx > 0, \int_0^1 x^{r+1}(1-x) \mid \varphi'(x) \mid dx < \infty \text{ for an } r \in \mathbb{R},$$

$$\alpha\varphi(x) + x\varphi'(x) \ge 0\ (0 < x < 1)\ \text{ for an } \alpha < r+1,$$
$$a \in C^{m+1}(\triangle_T) \text{ for an } m \ge 0,\ a(t,t) \ne 0\ (0 \le t \le T).$$

For instance, the Abel type integral equations of the first kind belong to this class.

Introduce the space $C^{m,r}$ of functions $u \in C^m(0,T]$ such that $\lim_{t\to 0} t^{k-r}u^{(k)}(t)$ exists for $k = 0,1,...,m$ and $\| u \|_{C^{m,r}} := \max_{0\le k\le m} \sup_{0<t\le T} t^{k-r} \mid u^{(k)}(t) \mid < \infty$.

Theorem. *Under formulated conditions, the inverse* $V_{\varphi,a}^{-1} \in \mathcal{L}(C^{m+1,r}, C^{m,r})$ *exists. Moreover,* $V_{\varphi,a}^{-1} \in \mathcal{L}(C^{m+1}[0,T], C^m[0,T])$ *in case* $r = 0$.

The proof [5,6] is based on the reduction of (1) to a second kind equation.

The convergence of polynomial and spline collocation methods for equation (1) is treated, cf. [2–4].

References

[1] G. Vainikko. Cordial Volterra integral equations 1. *Numer. Funct. Anal. Optim.*, 2009, 30, 1145–1172.

[2] G. Vainikko. Cordial Volterra integral equations 2. *Numer. Funct. Anal. Optim.*, 2010, 31, 191–219.

[3] G. Vainikko. Spline collocation for cordial Volterra integral equations. *Numer. Funct. Anal. Optim.*, 2010, 31, 313–338.

[4] G. Vainikko. Spline collocation-interpolation method for linear and nonlinear cordial Volterra integral equations. *Numer. Funct. Anal. Optim.*, 2011, 32, 83–109.

[5] G. Vainikko. First kind cordial Volterra integral equations 1. *Numer. Funct. Anal. Optim.*, 2012, 33, 680–704.

[6] G. Vainikko. First kind cordial Volterra integral equations 2 (in production).