Testing Covariance Structure in High Dimensional Setting

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Outline

HYPOTHESIS

Simple hypothesis: H_0: $\mathbf{\Sigma} = \mathbf{\Sigma}_0$, where $\mathbf{\Sigma}_0$ is known

- Covariance matrix is an identity matrix

- All parameters of $\mathbf{\Sigma}$ are fixed

Composite hypothesis:

- $\mathbf{\Sigma}$ is block-diagonal

- $\mathbf{\Sigma}$ has Kronecker product structure

- Not all parameters of $\mathbf{\Sigma}$ are fixed

LIKELIHOOD RATIO TEST

Denote:

- $X \sim P(\theta)$, where θ is r-vector

- $L(\mathbf{X})$ - sample likelihood

- Hypothesis H_0: $\theta = \theta_0$

Likelihood Ratio Test (LRT) statistic:

- $LRT = -2ln[maxL(\mathbf{X}|\theta_0)/maxL(\mathbf{X}|\theta)]$

- Under H_0 LRT is asymptotically $\chi^2(r)$

It has been shown that when the dimension of the covaraince matrix increases the likelihood ratio test will stop working correctly and will almost always reject the null hypothesis.

In order to deal with this problem, corrections to the test have been made so that it could be used in high-dimensional case as well (see Bai, Jiang, Yao and Zheng (2009) for an example).

RAO'S SCORE TEST

Due to Rao (1948)

Rao's Score Test (RST) statistic

- $RST = S(\theta_0)' I^{-1}(\theta_0) S(\theta_0)$ for simple hypothesis H_0: $\theta = \theta_0$

- $RST = S(\hat{\theta})' I^{-1}(\hat{\theta}) S(\hat{\theta})$ for composite hypothesis H_0: $H(\theta) = C$, where $\hat{\theta}$ is the MLE of θ, $H'(\theta) = (h_1(\theta), ..., h_t(\theta))$, $C' = (c_1, ..., c_t)$ and $t < r$.

- $S()$ - sample score function, $I()$ - Fisher's information matrix

- Under H_0 RST is asymptotically $\chi^2(r)$

Note: RST does not require θ to be estimated for simple hypothesis testing and does not require any estimation under alternative hypothesis.

RESEARCH HYPOTHESIS

Rao's Score Test works when $rank(\mathbf{\Sigma})$ is large and Likelihood Ratio Test does not work.

Simulation experiment was carried out in order to empirically test the hypothesis.

SIMULATION EXPERIMENT

The simulation experiment was carried out using R software and the following procedure:

1. Generate $X_1, ..., X_n$ from multivariate N_p distribution with mean vector μ and covariance matrix $\mathbf{\Sigma}_0$.

2. Use LRT and RST to test H_0: $\mathbf{\Sigma} = \mathbf{\Sigma}_0$.

3. Repeat N times, count the number of H_0 rejected by both LRT and RST.

4. Run for different $p = rank(\mathbf{\Sigma})$.

Figure 1. Simulation results: H_0: $\mathbf{\Sigma} = I$

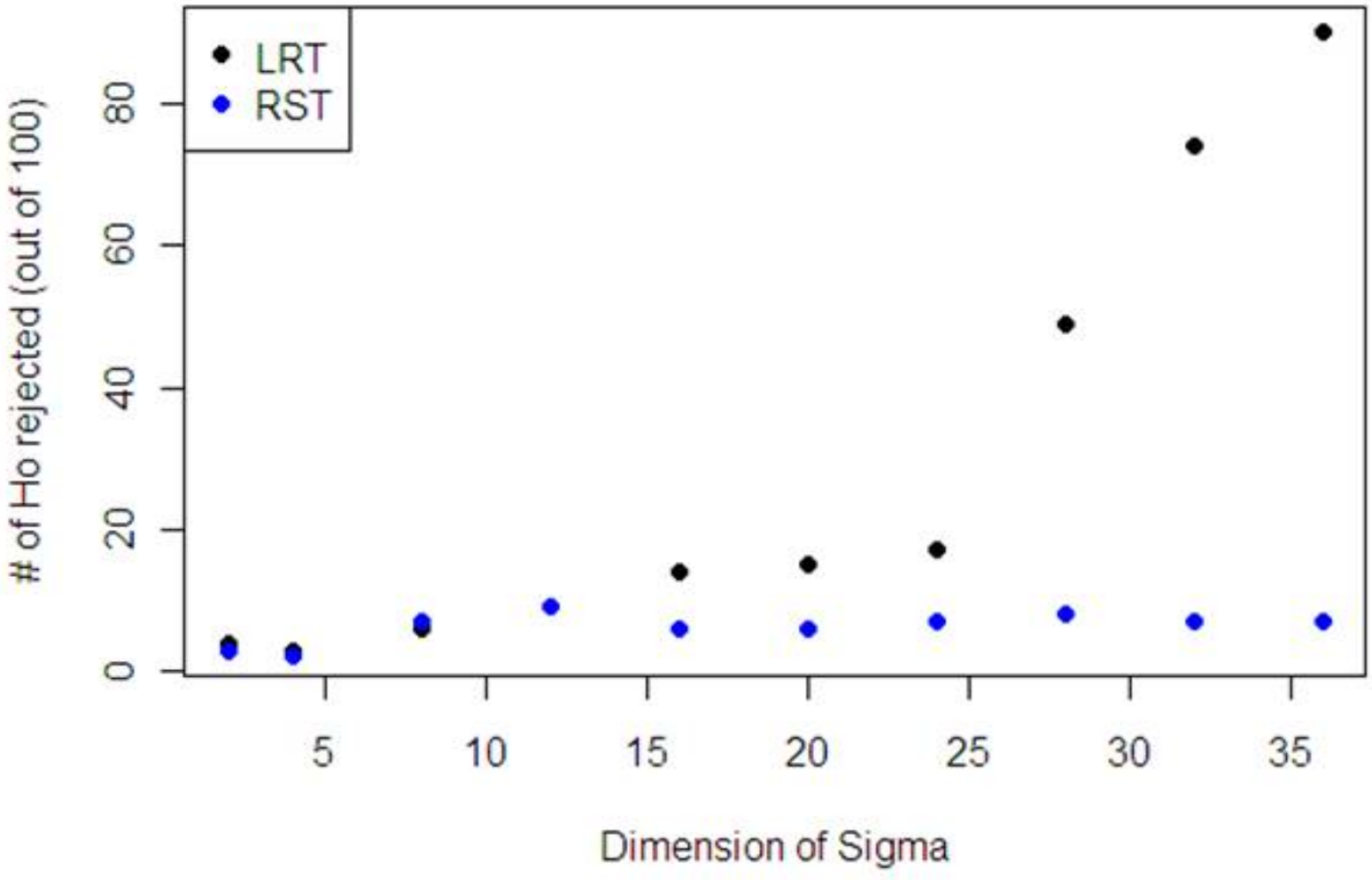
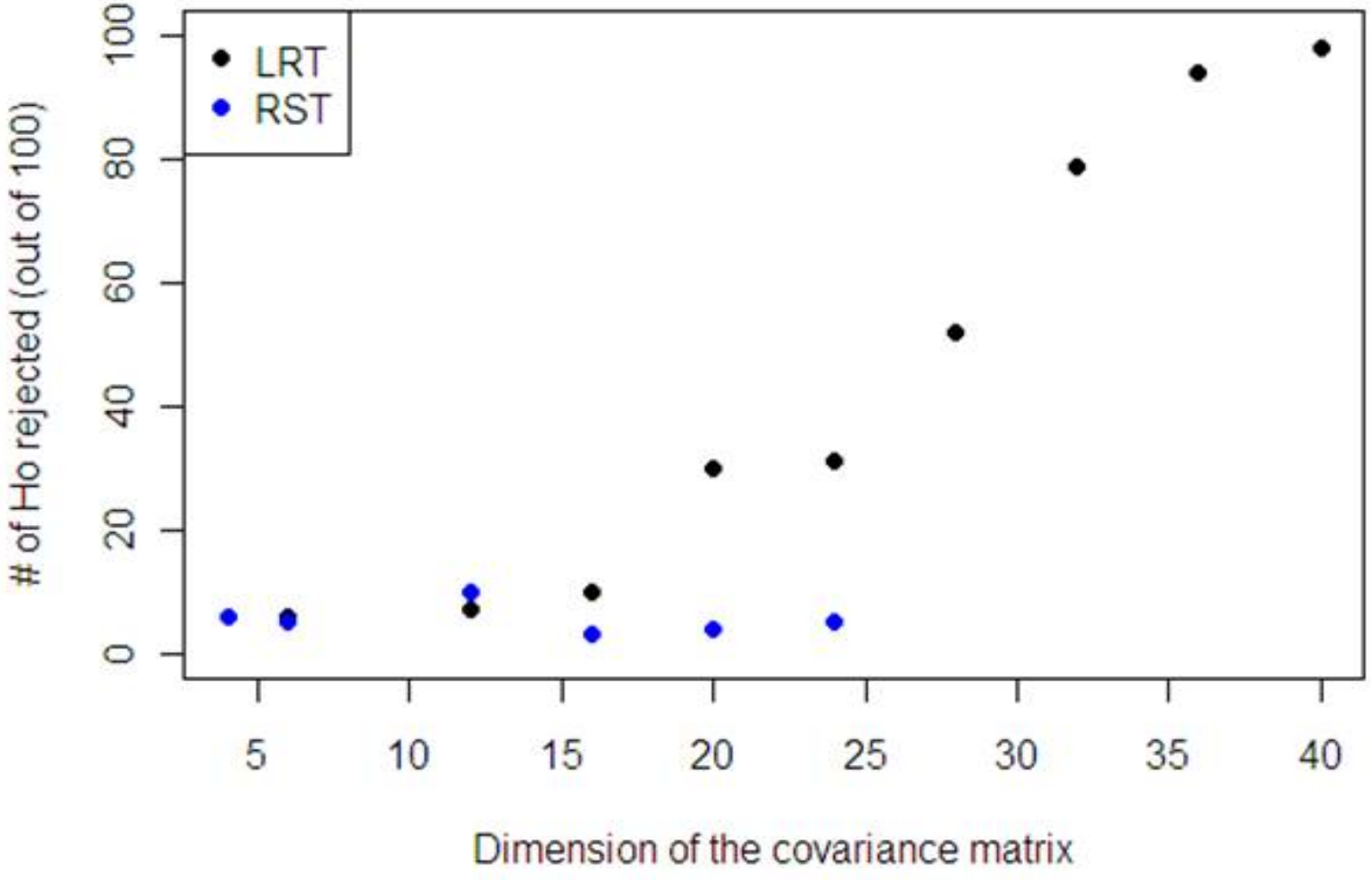


Figure 1. Simulation results: H_0: $\mathbf{\Sigma} = \Psi \otimes \mathbf{\Sigma}_0$



SUMMARY

Rao's Score Test should be preferred to Likelihood Ratio Test in high dimensional setting.

Future questions of interest:

- Comparison with Wald's Score Test,

- Simulations for different hypothesis types and sample sizes.

REFERENCES

Bai, Z., Jiang, D., Yao, J., Zheng, S. (2009), Corrections to LRT on large-dimensional covariance matrix by RMT, The Annals of Statistics 2009, Vol. 37, No. 6B, 3822-3840.

Rao, C.R. (1948), Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation, Proc. Cambrdige Philos. Soc., 44, 50-57.