

ON THE EQUIVALENCE RELATION IN STUDENTS' CONCEPT IMAGE OF EQUATION

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The concept of equation is, in principle, very elementary in mathematics. One can say that any mathematical expression of the form $A = B$, where A and B are relevant mathematical objects belonging to the same category, is an equation. However, a large variety of vague and incorrect conceptions about equation exists among mathematics teachers and student teachers. Only a minority of students can state a mathematically satisfactory definition for equation. (Attorps & Tossavainen, 2007a, 2007 b).

Seen from the point of view of mathematics and language of mathematics, a student's concept definition of equation can be wrong basically only for three different reasons: failure of understanding the equivalence relation $=$; misconceptions related to the truth value of the statement including the $=$ sign; or the confusion about choosing A and B from incompatible mathematical categories. With respect to mathematics at school, the latest reason is, nevertheless, only marginal.

We already have reported on the preliminary results on how teachers' and students' misconceptions related to the understanding of the properties of the equivalence relation are related to the misconceptions that teachers and students possess about equation (Attorps & Tossavainen, 2007a) and that the belief that the equation must always convey a true statement affects strongly how students themselves define equation (Attorps & Tossavainen, 2007b).

Since our original questionnaire did not completely reveal the relationship between the understanding of the mathematical properties of the equality relation and the concept of equation, we have collected new and larger data from Finland and Sweden (N=64) using a newly developed questionnaire to study this relationship and further to understand what kind of concept definitions students eventually possess about equation. As in the previous cases, we use a phenomenographic research method (e.g. Marton & Booth, 1997) in our analysis of the data. We also acknowledge the dual nature of mathematical concepts (e.g. Sfard 1991), the distinction of mathematical knowledge to the procedural and conceptual components (Haapasalo & Kadujevich, 2000), and the APOS theory (e.g. Asiala et al., 1997) when we classify students' concept definitions of equation and estimate how matured they are in a mathematical sense.

Students' conceptions of equation are often based on the existence of a variable to be solved and, generally, dominated by the operational/procedural view of the concept. Also, the expectation that an equation must always be a true statement was clearly revealed from our data. These conclusions are indicated e.g. by the fact that even 88% of the students claimed that $1 + 2 = 5$ is not an equation.

By our analysis, it appears that at least one third of the students do not understand the reflexivity of the equality. For example, 39% of students claimed that $x = x$ is not an equation. Also the failure of understanding symmetry of the equality is common. Half of all the students think that $x = 2$ is not an equation but merely "an answer to an equation", e.g. to $2 = x$. The classification of equations and their answers to different categories raises an immediate question: how well do these students understand the logic and the language of mathematics if they write $2 = x \Leftrightarrow x = 2$? The same phenomenon appears with the transitive property and with the similar generality: For example, 55% of the students claimed that $a = b = c$ is an equation and only a few of those who answered correctly motivated their answers by pointing out that there are several equations in the expression.

All in all, the misconceptions about equations which are related to the properties of the equivalence relation are surprisingly common among mathematics student teachers.

REFERENCES

- Asiala, M. Cottrill, J., Dubinsky, E. & Schwingendorf, K. (1997). The development of students' graphical understanding of the derivative. *Journal of Mathematical Behavior*, 16 (4), 399–431.
- Attorps, I. & Tossavainen, T. (2007a). *Is there equality in equation?* Paper presented at and to be published in the proceedings of CERME 2007, Fifth Congress of the European Society for Research in Mathematics Education, 17 – 21 February 2007 in Larnaca, Cyprus.
- Attorps, I. & Tossavainen, T. (2007b). *Is there always truth in equation?* Paper to be presented and published in the proceedings of NORMA 08, Fifth Nordic Conference on Research in Mathematics Education, 21 – 25 April 2008 in Copenhagen, Denmark.
- Haapasalo, L. & Kadjevich, Dj. (2000). Two types of mathematical knowledge and their relation. *Journal für Mathematik-Didaktik*, 21 (2), 139–157.
- Marton, F. & Booth, S. (1997). *Learning and Awareness*. Mahwah, N.J.: Law Earlbaum.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22, 1–36.