

# **DETECTING MATHEMATICAL ABILITIES IN STUDENTS' SOLUTIONS OF MATHEMATICAL PROBLEMS**

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Mathematical talent appears to be constituted by a spectrum of abilities that each individual displays in different degrees. Identifying and classifying the mathematical abilities is a major task, and V.A. Krutetskii has done a thorough study in this field in the Soviet Union in the middle of the last century (Krutetskii, 1976). Krutetskii claimed that mathematical ability could be looked upon as divided into seven categories. Those abilities are shown to be more abundant in capable students than in others. His technique was to design mathematical test items addressing these abilities one by one, and use factor analysis on the students' responses to justify his claim.

While Krutetskii uses different sets of problems to address the specific abilities and thereby justifying his system of abilities, we will do it the other way around. Our purpose is to give the student a set of rich mathematical problems, and by analysis of written solutions and interviews of the students, we try to detect what abilities the students display.

The design of problems is a crucial point in our study. To meet Krutetskii's categories of abilities our problems must apply to the students' creativity and flexibility in thinking, and also give them good opportunities to express generality in their reasoning. This leads us to look at so called rich mathematical problems as candidates for our test problems. Different researchers have used the term "rich mathematical problems" in a partly different meaning, but we have found Hedrén's et al. definition close to our demands, (Hedrén, Hagland, Taflin, 2005; Taflin, 2007). Tasks that are supposed to stimulate students to formulate generalizations can be found in various works treating mathematical problem solving. One such work is the book by Mason et al. (Mason, Burton, Stacey, 1985).

To collect data, an extended task sheet with three or four selected problems is used.

These problems are of two types: 1. Problems to be worked out individually by the students in the classroom during a limited amount of time. 2. More complicated problems to be mulled over during a period of two or three weeks. To promote the reliability of the investigation, data from this latter type of problems is supplemented by a clinical interview with the

problem solver. Classroom tasks have been made for full classes on five occasions, both with upper secondary school students and first year mathematics teacher students. A few of these students also have completed the long – term tasks, but this far no interviews have been conducted. We expect, however, more students to complete the long-term tasks in the near future.

The research question in focus is:

*By looking at the outcome from students' solving rich mathematical problems, which of Krutetskii's mathematical abilities can be identified, and how are these abilities revealed?*

In the analysis of the results we first look at each problem in the light of the Krutetskian scheme and make our own interpretation of that in the context of that specific problem. We then look at the students' solutions in order to detect which of these abilities they use in solving the problem.

It is a well-known fact that working with challenging problems foremost attracts high achieving students. For these students problem-solving activities might meet their needs of more adequate education in mathematics. We believe, however, that with properly designed tasks, it is possible to fruitfully work with problem solving in a mixed-ability classroom. Knowledge about the abilities that a certain problem might reveal, can be used as a parameter when classifying problems into some kind of taxonomy. It may also increase our understanding of the nature of mathematical talent.

References:

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