Spectrums of knowledge types – mathematics, mathematics education and praxis knowledge

Håkan Lennerstad,
Blekinge Institute of Technology

Abstract

Knowledge in different research paradigms is discussed: mathematics, mathematics education and a paradigm of practical knowledge. I argue that the three paradigms as highly distinct, different and important. They try to cope with entirely different types of knowledge, all highly relevant for mathematics teachers.

While mathematics is deductive and mathematical education is evidence based, practical knowledge is a type of knowledge that professionals in any profession develop by experience and by exchange with other professionals. It is based on experience more than on written text. It is well known that it to a large extent is difficult to articulate. Such knowledge is also essential in important types of mathematical knowledge. We discuss the role played by vagueness in mathematics. We also discuss linguistic mathematics knowledge which typically is present but mostly unformulated, as the mother tongue.

I argue that mathematics, pedagogy and mathematics education suffers from drawbacks by being strongly rooted in the positivist tradition, in which knowledge can always be expressed in words – otherwise it is not knowledge. Central aspects of teacher's day-to-day profession are too complicated to be captured in words. However, work has been done to allow such practical knowledge to be formulated among professionals. I would like to sketch a more fluent cooperation between the paradigms, in which the advantages of all the different knowledge types may interact and become increasingly useful to each other.

For such an idea to reach reality, an efficient meeting form is needed. The Dialogue Seminar is developed precisely to study and communicate difficult-to-articulate practical knowledge among experienced professionals from different areas, using analogue and metaphor as catalysts. This offers mathematicians, mathematics education researchers, mathematics teachers and teacher students, and others, an excellent opportunity to listen in depth to each other, and to have a dialogue.

Introduction

In order to improve mathematics education, are our efforts well spent? Are there possibly other ways of work that may lead to more significant improvements? This is an extremely basic issue for an applied science, which lies at the bottom of this paper. It is a question that directly concerns how three professions form and perform their work: mathematics education research, teacher education, and teaching.

This paper tries to use a viewpoint of the teacher profession as a profession among professions. When comparing to non-teacher professions, one particularity of the teacher profession stands out: its relation to *knowledge*. It has particularly strong relations to knowledge in two ways. The first is that its main purpose is the learning of certain subject knowledge for other humans. The second is that there is a long knowledge tradition about how teaching and learning can be done: pedagogy and didactics – in later years promoted to education science. Knowledge is essential in all professions, but for teachers it is the very material of work, and there is a lot of written knowledge on how this particular profession works – since the school is so important in society.

This strong knowledge tradition concerning the teacher profession has been developed in a positivist tradition – which was virtually the only tradition in the previous century. In this tradition, knowledge is what can be formulated in words (Johannessen, 2006B, p. 273), a type of knowledge that can be called *propositional knowledge*. This viewpoint neglects much of practical knowledge, a fact that is not changed by propositional knowledge that has practical knowledge as focus (such as partly this article). Practice is different in essence from propositional knowledge. For example, it is very much possible to be an expert in every possible aspect of listening dialogues, without being able to take part in one. Another person may be known to reoccurring valuable dialogues with students, but unable to describe what a dialogue is and how it works in depth.

Despite the strong knowledge tradition in the teacher profession, difficult-to-articulate practical knowledge is important in all professions – also in the teacher profession. Practical knowledge enables a professional to act in appropriate ways in unforeseeable teacher situations – it develops intuition. It is a knowledge that enables skill – the ability to *perform* the profession. This ability is distinct from knowing.

In terms of action research, for example, pedagogy and mathematical education are searching ways to handle teachers' reality and practical knowledge. However, it is not easy to allow teachers' to fully express their views of their teaching situation if it contradicts established paradigms. This problem is one main focus of this article. The philosophical foundations of practical knowledge (Johannessen, 2006A) may also contribute to the understanding of propositional knowledge, both its limits and its values.

A different but related serious problem for research in mathematics education is that it serves two goals that are rather conflicting. One is to help teachers; the other is to fulfil the requirements for research. The second goal is a long term goal that makes results more reliable, but tends to make results inaccessible for teachers. In effect, a researcher must choose between reaching teachers or reaching researchers. Some ambitious researchers have written popular versions of their research, more accessible for teachers and decision makers. This should be mandatory for an applied science. We need to leave behind the fact that "popular" writing sometimes is seen as negative among researchers.

The purpose of this paper is to find ways to discuss and illuminate the non-propositional components of mathematical and mathematics education knowledge, which take many different forms. It is also to suggest ways in which the knowledge traditions can cooperate, develop and stimulate each other. For this, the professional reality experienced by mathematics teachers need to be a well developed starting point, from which mathematics education research provides a resource.

Differences between paradigms, research questions

A Dialogue Seminar is an organized dialogue between professionals, to be described later in this text. By taking part in Dialogue Seminars with other teachers, a rather obvious observation has become clear to me. It is that a teacher professionally faces a complex teaching situation with many different parts to be handled well. Such parts are subject knowledge, how to present subject knowledge, to understand students present level of subject knowledge, how to respond to students questions and actions, correspondence to neighbouring subjects, how to plan future lessons, etc. etc. These parts need not only to be handled well; they also need to be balanced into a reasonable whole. I argue that from an epistemological viewpoint, mathematics education generally provides solid information about one or a few of the different parts at a time, but rarely addresses the balancing act that a teacher needs to handle. On the other hand, from the viewpoint of practical knowledge, the balancing act naturally attracts focus, since here active teachers formulate their needs and views. On the other hand, in the praxis paradigm solid evidence based results are rarely produced, as is more typical for mathematics education. Results in mathematics education are usually more solid and general. We have a trade-off between generality and authenticity.

These two paradigms have different and complimentary roles to play. In praxis research, a group of teachers plays the main role in problem formulation as well as in reformulation and development. Mathematics education research results are founded in evidence and theory, which to some extent limits which problems that may be studied. Mathematics education results tend to be more reliable, less dependent on local culture; however the depth typically means a larger distance from teachers' experiences. If we compare with mathematics, one may claim that the overwhelming reliability of mathematics lies in that here extremely limited problems are studied – so limited that they allow a very high degree of certainty.

Following a line of praxis knowledge, I argue that there are fundamental categories of knowledge that are vital for teachers and that cannot be accessed by traditional analytical approaches. The Dialogue Seminar can be seen as a respectful and well developed means, mainly using analogy, metaphor and dialogue, often taking advantage of areas as history, philosophy, mathematics education, to put practical knowledge in motion that is particularly useful for teacher's education, and that provide an answer to the following fundamental question:

How can the sources of knowledge and skill that experienced teachers possess become available for teacher students, as described by experienced teachers themselves, and in ways that teacher students find valuable, supported by mathematics education results?

This is one of the main research questions in this paper. Related questions addressed here are:

Which types of knowledge are relevant in the mathematics teacher's profession? Which types of knowledge are important in subject knowledge in mathematics? How can the different knowledge types be handled in successful ways?

The purpose of the paper is to put forth underestimated types of knowledge, to give a general view of the epistemological landscape, and to suggest ways to design this landscape. This proposed design is to professionally take advantage of existing experience by allowing different knowledge traditions to meet systematically and constructively. Teachers will not acknowledge the value of their own resources of experience unless researchers emphasize these resources and try to find ways to develop them.

Culture, paradigms, and knowledge

Initially, I sketch a few starting points. The main one is that all knowledge lives in a certain culture, and it is important to recognize the cultural borders, partly to be able to surpass them. Any piece of knowledge has a certain "home culture", outside of which is false or meaningless. Certainly, boundaries between cultures are both complicated and fuzzy, not to say the culture concept itself.

Also mathematics is strongly cultural. To be able to judge whether a mathematical statement or proof is valid or not, you need to pass through many years of mathematics education, and you need to do so in a way that is acceptable for the mathematics teachers/researchers community. Effectively, you cannot dispute a mathematical statement with any credibility without first becoming an accepted member of its culture. Many scientists are convinced that mathematics is a true property of reality in the sense that any intelligence will find essentially the same mathematics as humans have found. If testing and disputing is essential to scientific truth, this conviction is not very scientific, since the statement has never been tested by any non-human. Of course, convictions not always need to be scientific. But this conviction, unprovable but accepted as if it was proved, may be an example of what Poul Ernest calls "the ideology of mathematics" (Rowland, 1999, p. x (in foreword)).

On the other hand, mathematics is a meta-subject in the sense that it concerns quantitative reasoning in *any* area. One may say that mathematics summarizes and formulates such quantitative thought that is not contradicted in any domain of knowledge, including non-scientific domains. As such it has the unique property of being tested by other sciences. This is a test that mathematics has passed with brilliance, leading to a sometimes overwhelming admiration. The success of mathematics turns the attention towards the subject itself with renewed curiosity: perhaps particularly towards its apparent abstractness and the strangely non-obvious meaning of its symbols and their praxis of manipulation.

Cultural divides – an ever present problem

Cultural divides may of course cause misunderstandings – small or large. Cultural divides are everywhere in society. We all need to be able to reach out to a more or less different culture than the one we belong to, or are most comfortable with. Teachers need to be able to communicate with pupils for the teaching to be successful, and thus need to understand the culture of pupils.

Also the cultural divide between mathematics education researchers and mathematics teachers must be handled well. Of course, mathematics education needs to find applications in practical classroom activity. Anna Sfard describes in (Sfard, 2005) mathematics education researchers' changing attitude towards teachers. She found that the focus has moved from students towards teachers (ibid., p. 397), only a few years ago. Concerning teachers, she writes (ibid. p, 401):

> *... the teacher is described as an ally, a kindred spirit, a partner, a colleague. This egalitarian self-positioning toward the teacher is a rather dramatic change in the research discourse which, only a few decades ago, was imbued with patronizing undertones. Today, the researchers stress that their studies are done* ***with*** *the teachers rather than* ***about*** *her.*

Propositional and professional knowledge

In (Hudson, 2002), Shulman's (Shulman, 1987) model of categories of knowledge of teachers' knowledge is used. It contains the following categories, where I in parenthesis have added counterparts/characterizations relevant for the discussion in this paper. It illustrates well the balancing act that teachers constantly face in their profession:

- Knowledge of subject matter (mathematics)
- Pedagogical content knowledge (mathematics education)
- Knowledge of other content (knowledge of the educational program)
- Knowledge of the curriculum (course knowledge)
- Knowledge of learners and their characteristics (student culture)
- Knowledge of educational aims (political and school knowledge)
- Knowledge of the educational context (school culture)
- General pedagogical knowledge (pedagogy, classroom management)

Parts of these knowledge categories can be formulated in words, a knowledge type that may be called *propositional knowledge* (Göranzon, p. 19). Two types of knowledge that cannot easily be expressed in words are also part of most of these categories. These are *practical knowledge*, which is knowledge that contains experiences from having been active in a practice, and *knowledge of familiarity*, that is built by interaction with collegues about examples of practice. These two may together be called *professional knowledge*. This is knowledge with special properties, described by Kjell S. Johannessen (Johannessen, p. 229) as follows:

"Professional knowledge is essentially characterized by two basic traits: (a) It is acquired over a relatively long period of time by individuals; and (b) attempts as articulating it in some reasonably satisfactory way all fall short of even elementary standards of plain speech."

These two properties disqualify professional knowledge as knowledge from a positivist standpoint. In a positivist view, knowledge that is individual is not knowledge. Furthermore, knowledge that cannot be articulated is not knowledge. Here are Johannessen's perhaps more nuanced words (ibid.):

"Both of these traits stand out as inherently provocative to the adherent of the received and positivistically tinged view of knowledge that is predominant in our time. The first trait threatens to make knowledge dependent on individuals; and the second more than indicates that some kinds of genuine knowledge may in basic respects be resistant to verbal or notational articulation and thus be beyond the reach of language."

Is such professional knowledge, difficult to formulate and perhaps to study, important for mathematics teachers? Well, a famous and experienced research mathematician has once claimed:

Logic is very important in mathematics, but it has never been used to find a proof.

Mathematics students attempt to solve mathematical problems, which is a counterpart to mathematician's search for proofs. Inherent in the statement is the recognition of the vague concept of "intuition", which is addressed by Davies and Hersch (Davies & Hersch 1995, p. 435) in the following way:

(1) All the standard philosophical viewpoints rely in an essential way on some notion of intuition.
(2) None of them even attempt to explain the nature and meaning of the intuition that they postulate.
(3) A consideration of intuition as it is actually experienced leads to a notion which is difficult and complex, but it is not inexplicable or unanalyzable. A realistic analysis of mathematical intuition is a reasonable goal, and should become on of the central features of an adequate philosophy of mathematics.

These are strong words about the role in mathematics on something so vague and undefinable as "intuition". Personally, I look forward to the "realistic analysis of mathematical intuition". Probably a very metaphoric and poetic language, far from traditional mathematical language, is required to develop this analysis – which would shed light on this very language.

Vagueness and knowledge

Epistomology and linguistics are firmly related to mathematics education, and have during later years found increasing attention. This is described by Paul Ernest in the preface of (Rowland, 1999, p. x (in foreword)). He describes that the lack of attention to linguistics may depend partly on the focus of mathematical thought over talk. He continues to write that it may also depend on absolutist epistemology of mathematics, in the light of which language serves to describe absolute logic. Spoken mathematics is imprecise and has limited value in this perspective. This diverts the attention from students' actual mathematical thinking.

On the importance in mathematics of the opposite of preciseness, vagueness, Ernest writes in the same preface to (Rowland, 1999):

> Precision is the hallmark of mathematics and a central element in the "ideology of mathematics". Tim Rowland, however, comes to the startling conclusion that vagueness plays an essential role in mathematics talk. He shows that vagueness is not a disabling feature that detracts from precision in spoken mathematics, but is a subtle and versatile device which speakers deploy to make mathematical assertions with as much precision, accuracy and confidence as they judge the content and context warrant.

Thus, vagueness need to be restored as a valuable compliment to precision for good mathematics learning. Certainly, vague descriptions may lead to misinterpretations, but that is also possible for precise descriptions. Essential is that descriptions are to the point, and that the teacher has an idea of how students interpret. A better dialogue is required to understand each others interpretations – vague or not.

Here are two cases of vagueness in mathematics that I intend to discuss:

1. Vagueness that occur when teachers translate mathematics formulae into English (or other natural language) – a vagueness that reveals underlying webs of meaning that otherwise go unnoticed. Translations require a certain linguistic view of mathematics.

2. Vagueness in knowledge that reside in mathematics teachers' professional experience.

A translation "$f(x) = x^2 + 1$" (symbolic language) into "The function f squares a number x and then adds one." (same statement in English) might be translated differently by another teacher. This difference may by teachers be seen as a problem, but it is actually the converse of a problem.

Such discrepancies reveal for students the realms of knowledge that underlies a formula. Each and every important concept in any subject has several different uses, associations and meanings as seen from different angles. This is equally true for mathematical concepts, but it becomes rarely visible – mainly during such "problems".

The differences in interpretation visualize the webs of knowledge hidden behind formulas, which then becomes learnable. Formulas express exactly what we agree about, which may be too general for students to relate to. Translating formulae reveals for students a vagueness that is part of any concept, without which the knowledge is crippled or incomplete. We need to teach also vague knowledge. In the vagueness of teachers personal mathematical interpretations may mathematical meaning grow and prosper (concerning linguistics of mathematics, see Lennerstad, Mouwitz, 2004).

Intuition, syntax and semantics of mathematics

In (Lennerstad, 2005), five types of mathematical knowledge are rather precisely defined in relation to the symbolic language (from $1 + 1 = 2$ to research formulae), for which the term "Mathematish" is practical from a linguistic viewpoint. These are

1. Knowledge *above Mathematish* – knowledge in *how to use the rules* of Mathematish in order to solve problems and find proofs. This is very different from knowing which rules exist, and it consists mainly of *intuition*. Not much is

formulated explicitly. The famous book *How to solve it* by George Polya, written 1945, (Polya 1988) is virtually the only book in the area. By this knowledge a person decides how to try to solve a problem. Computer programs do badly in this area.

2. Knowledge *in Mathematish* – knowledge of Mathematish rules. This is a finite and distinct set of rules (although there are exceptions, synonyms, double meanings and context dependence as in any other language), and is computer programmable. However, this knowledge is mostly tacit knowledge for mathematicians and mathematics teachers, just as the mother tongue for any natural speaker. Explicit knowledge is unnecessary (except perhaps for teaching), so it is rudimentary.

3. Knowledge *below Mathematish*. Here we have semantics of mathematical formulae. While natural languages have one type of semantics, we may distinguish three types of semantics for mathematics. The formalized logic and the generality produce two distinct types that are characteristic for mathematics:

3a. *Numeric/formal semantics*. Exactly this type of semantic is under scrutiny in a proof. It may give certainty about what is true or not, but conceptual semantic (3c) is more related to a sense of understanding (see Sierpinska, 1994).

3b. *Applied semantics*. Since mathematics is general, one may often convince oneself of a mathematical statement by consideration of a familiar application. Each application defines one type of meaning of a mathematical formula.

3c. *Conceptual semantics*. This type of semantic is neither formal nor connected with any particular application, though it is certainly not isolated from any of these. It corresponds mostly to the semantic that natural languages possess, and it strongly related to an experience of understanding. One main source of conceptual semantics of Mathematish is geometry, which obviously is not necessarily application-specific. This semantic probably develops in dialogue about relations between formulae, geometry, applications, numerics, logics, events during mathematical activity and personal associations and reflections.

Certainty versus silence

One credo of serious research may be: *Of what we are not certain, we remain silent.* This may be characteristic of scientific ambition – to avoid to be falsely interpreted. However, if scientific ambition increases, this would imply that the epistemological domain of certainty shrinks, and the domain of silence grows according to the credo. This is a serious problem for scientific areas where there exist essential types of knowledge that cannot be formulated or described with certainty. Results with high certainty may turn out to be of minor importance in comparison to other forces that vastly affect the situation.

An applied science needs to target not only what can be studied with certainty, but also uncertain features may be considered important. The alternative is to drift towards irrelevance within the borders of a self chosen perfection.

A praxis paradigm

In the tradition of the Dialogue seminar there is not much fear of vagueness. Instead, the unformulated knowledge that is possessed in a group of experienced professionals is focused. Unsuccessful computerizations of workplaces in the 80-ies were a starting point of this line of research. Reforms were related to a conviction that most or all of the relevant knowledge could be programmed in computers. Instead, groups of experienced professionals possess more knowledge that they are able to formulate in words. I have earlier described the knowledge categories *practical knowledge*, *knowledge of familiarity* and *propositional knowledge*. The two first develop while taking part in a professional practice, and interaction with colleagues, respectively. Such knowledge is not necessarily individual; it usually lives in a professional group. Sometimes the group is needed to find an appropriate action – for the knowledge to come alive.

The dialogue seminar is an arena for professional groups to find, formulate, characterize, stimulate and value their practical knowledge based upon experience, or in some sense (not necessarily with words) make it palpable or present. Analogue and example are important elements. Historical texts are often rich in these respects. Musicians, engineers and others may participate in the same sessions. Meetings with other professions incur no rivalry, and appear surprisingly often to be fruitful for all parts. The sessions work with writing as a method of reflection. All members prepares actively each session along a certain theme with a text to be read aloud and reflected upon. Then, each member is invited to comment verbally upon each text that is read.

The invitation to reflect from experience is central. It makes the sources of experience increasingly visible for each bearer of that experience. These sources may grow into resources of knowledge that deliver more and more. Associations to other persons experiences, which may be partly similar and partly different, is the tool for this discovery.

The dialogue seminar is an arena where mathematics teachers, mathematics education researchers, mathematics teacher educators and mathematicians can meet, listen to each other in depth, and learn from each other through dialogue. It appears as if mathematicians often experience dialogue with musicians as particularly valuable. This may come from the fact that intuition is important both in music and in mathematics, in somewhat similar meanings, as described above in (Davies & Hersch 1995), while musicians may have advanced longer than mathematicians in formulating their intuition.

Göranzon and Hammarén (Göranzon, Hammarén, 2006) describe the major goals of the dialogue seminar as follows:

The dialogue seminar method is a method of working that aims to
(i) create a practice of reflection
(ii) formulate problems from the dilemma
(iii) work up common language
(iv) train the ability to listen.

I remark that the ability to listen does not follow automatically from the ability to talk. In academia, the ability to talk is trained much more than the ability to listen. We learn in three ways: from reading, one-way-listening during lectures, and dialogue and reflection with others. Self-reflection is of course always a component. Reading is a form of listening, but without an opportunity of dialogue. The relative dominance of these learning modes in academia determines the corresponding degrees of training, and which abilities that develop.

Teaching and *Bildung*

Hudson also describes differences between the Anglo-American curriculum tradition versus the German. In the first case the teachers are employees of the school system which has a strong formal control of teachers (Westbury). Professionalism is achieved by *training* and *certification*, to teach the curriculum. In the German tradition, the teacher is directed rather than controlled by the institutional framework. There is a larger professional autonomy for the teacher, for example in interpretation of the curriculum. This is related to the presence of the idea of *Bildung*. Klafki (Klafki, 1998) has specified three main elements of *Bildung*: (i) *self-determination*, to be enabled to make independent responsible decisions, (ii) *co-determination*, to be enabled to contribute together with others to the society, and (iii) *solidarity,* actions to help others.

Khalid El-Gaidi's doctoral thesis (El-Gaidi, 2007) *Teacher's professional knowledge – Bildung and reflective experiences* (my translation of the title, which was in Swedish only) was defended 2007 at the Royal University of Technology in Stockholm. Here teachers' skill at a technical university is examined. The dissertation starts with a case study in form of a Dialogue Seminar where university teachers participated actively in a series of meetings about their view of their praxis and of skill. The discussion on knowledge and skill based on this case study converged clearly towards *Bildung*. In this thesis *Bildung* is seen from many aspects, such as the ability to see the limits of the activity and to avoid misunderstandings. It is also seen as a standpoint involving judgement, sensus communis and taste (Gadamer 2004), a way to view knowledge in that we need to recognize the different forms of cultures we live in, and as a way of seeing the whole picture, intuition and as rhythm of thought and communication.

A course in university pedagogy during the fall 2006 at the Blekinge Institute of Technology was called “Mathematics in engineering education”, and applied the Dialogue Seminar method. The participants were telecommunication and mathematics teachers. Mathematics education texts were studied and reflected upon, providing the basis for the dialogue. The teachers found that the dialogue visualized and synchronized common experiences and conclusions from the practice of mathematics teaching in ways that otherwise happened rarely or never.

Conclusions

Like in all professions, a large part of the relevant mathematics teacher knowledge is difficult-to-articulate practical knowledge. This is in striking parallel with mathematical subject knowledge in the sense that also this knowledge has important vague and tacit components. This is largely underestimated, partly since they come into play mostly when mathematics is verbalized. Mathematics is still a subject with a low degree of dialogue.

Simultaneously, there is a division between mathematics teacher’s culture and mathematics education researcher’s culture that is dangerous from both perspectives: for improvement of mathematics teaching, and for the relevance of mathematics education research. This division stems from the differences in aims and history. A complementary way to work is proposed in this paper to bridge divisions. A tool for this is the Dialogue Seminar, which is designed and developed for the purpose of allowing experienced professionals to search and find difficult-to-articulated types knowledge – or at least to find tangible knowledge resonances. Such seminars are verbal in nature, but works with both reading and writing as reflection and individual preparation before the seminars. The proposal is to apply this method for the benefit of mathematics education. This offers mathematicians, mathematics education researchers, mathematics teachers and teacher students, and others, an excellent opportunity to listen in depth to each other, and to have a dialogue.

References

Davies P., Hersch R, (1995). *The mathematical experience,* Birkhäuser, Berlin.

Gadamer H.G. (2004). *Truth and method,* London: Continuum Publishing group.

Göranzon B., (2006). The Practice of the Use of Computers: A Paradoxical Encounter between Different Traditions of Knowledge, in *Dialogue, Skill & Tacit Knowledge,* Göranzon B., Hammarén, M., Ennals R. (eds), Wiley’s.

Göranzon B., Hammarén, M., (2006). The methodology of the dialogue seminar, in *Dialogue, Skill & Tacit Knowledge,* Göranzon B., Hammarén, M., Ennals R. (eds), Wiley's.

El-Gaidi, K., (2007). *Lärarens yrkeskunnande – bildning och reflekterande erfarenheter*, Dialoger, Stockholm.

Hudson B., (2002). *Holding complexity and searching for meaning: teaching as an reflective practice,* J. Curriculum Studies, Vol. 34, No. 1, 43-57.

Johannessen K., (2006A). Knowledge and Reflective Practice, in *Dialogue, Skill & Tacit Knowledge,* Göranzon B., Hammarén, M., Ennals R. (eds), Wiley's.

Johannessen K., (2006B). Rule Following, Intransitive Understanding and Tacit Knowledge: An Investigation of the Wittgensteinian Concept of Practice as Regards Tacit Knowing, in *Dialogue, Skill & Tacit Knowledge,* Göranzon B., Hammarén, M., Ennals R. (eds), Wiley's.

Klafki W. (1998). Characteristics of critical-constructive Didaktik. In B. B. Gundem and S. Hopmann (eds), *Didaktik and /or Curriculum: An International Dialogue,* New York: Peter Lang 307-300.

Lennerstad H. & Mouwitz L. (2004). *Mathematish – a tacit knowledge of mathematics*, In C. Bergsten & B. Grevholm (Eds.), *Mathematics and language. Proceedings of MADIF4* (pp. 169-184). Linköping: SMDF.

Lennerstad H. (2005). *Matematikens dubbelnatur – självtillräckligt språk och undflyende innehåll*, Utbildning och Demokrati, vol. 14, nr 2.

Polya, G. 1988. *How to solve it* Princeton Univ. Press.

Rowland, T. (1999). *The pragmatics of mathematics education, vagueness in mathematics discourse,* Studies in Mathematics Education 14, Routledge.

Sfard, A. (2005). What could be more practical than good research? *Educational Studies in Mathematics*, 58, 393-413.

Sierpinska, A. (1994). *Understanding in mathematics*. London: Falmer Press.

Schulman, L. S. 1987. Knowledge and teaching: foundations of the new reform. *Harward Educational Review.* 5 (1) 1-22.

Westbury I., (2000). Teaching as a reflective practice: what might Didaktik teach curriculum?, In Westbyry, S. Hopmnann and K. Riquarts (eds), *Teaching as a Reflective Practice: The German Didaktik Tradition (*Mahwah, NJ: Erlbaum), 15-40.