

Reading word problems

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Students' work on word problems place demands on both comprehension and solving strategies. When reading a word problem, students construct a mental representation of the problem text that serves as the basis for solving the problem. Still reading and solving word problems is not necessarily a linear process. Students might for instance reread the problem text during solving or when evaluating answers. This paper outlines a framework for investigating how strategies for reading and solving word problems are connected before presenting two examples of reading strategies demonstrated by students giving verbal protocols.

Introduction

Word problems are not a novelty in school curricula. Different aspects of student work on word problems are extensively researched, still many students, even some teachers, find word problems challenging. Much research is conducted within a cognitive framework, researching different aspects of reading or solving word problems (Cook, 2006; Cummins, Kintsch, Reusser & Weimer, 1988; Reed, 1999; Verschaffel, Greer & De Corte, 2000). In the study *Understanding and working on word problems* I am concerned with students' strategy use while working on a group of word problems. The overall aim of the study is to describe different levels in strategy use and domain competence. This paper outlines a framework for investigating students' strategy use while reading and solving word problems. Two examples of raw data are presented and discussed in the last section of the paper.

Prior research on students' comprehension of word problems

Word problems are defined as verbal descriptions of problem situations by Semadeni (1995). Each problem embeds one or more questions that can only be answered by first constructing an understanding of the mathematical relationships in the text. They are traditionally associated with a school setting wherein the student is being asked to solve the problem in connection to a mathematics lesson, a test situation or as homework (Verschaffel et al., 2000). When students do not try to solve an item in mathematics, or arrives at the wrong solution,

many of them do so due to lack of understanding of parts or the whole of the item context (Reed, 1999). To be able to solve a word problem a student must identify the question embedded in the text, choose an appropriate solving method and conduct the necessary calculations. Hopefully the student evaluates the answer (s)he arrives at as well.

While students' competence traditionally is evaluated through inspecting calculations and explanations, causes for student' difficulties or errors could also lie in shortcomings in text comprehension or difficulties in constructing a mental representation of the underlying mathematical situation (Reed, 1999). Cummins et al. (1988) found that solution performance is related to comprehension of the word problem text and to the language in the text. When students make errors and arrive at a wrong solution, it is often a correct solution of the word problem the student thinks (s)he is solving. In other words they solve correctly the problem as it is constructed in their mental representation of the problem text.

My research interest lies in investigating students' strategy use when reading and solving word problems. This is not necessarily a linear process; students might reread problem text while planning or solving (Hegarty, Mayer & Monk, 1995). Findings in a pilot study also suggest that reading and solving strategies are intertwined: Strategies for reading¹ were visible also when students were working on solving the problems (Nortvedt unpublished work).

Word problems are traditionally short and condensed texts. The pilot study revealed that texts are often *so* short that students keep necessary numbers in working memory and move directly on to solving the problem. This was the case both for comprehended as well as for misunderstood problems. When working on their solutions, students revealed several general reading strategies such as re-reading, looking for key words, clarifying etc (ibid.).

When reading the reader constructs a mental representation of the text (Bråten, 1994). Nathan, Kintsch & Young (1992) suggest that when reading word problems, students construct a qualitative situation model representing the social context of the word problem and a quantitative model containing the algebraic structure or schema of the text.

Thevenot, Devidal, Barrouillet & Fayol (2007) argue that this mental model is a situation model containing the mathematical relationships, not an activated schema. This suggestion is based on increases in success rate for word problems when the question is placed in front of the body text versus after as in traditional word problems. The situation model is a *temporary structure stored in working memory that contains in addition to the*

¹ This refers to reading comprehension, not decoding

mathematical information necessary to solve the problem, nonmathematical information that is related to the context in which the situation described by the problem takes place. A situation model is therefore more qualitative and less formal than a schema (ibid p. 45).

Prior research on word problems has revealed that addition and subtraction problems can be placed in different categories according to semantic structures in the text. If the content of the word problem is made more explicit without changing the semantic structure, the number of students solving the problem successfully will raise (De Corte & Verschaffel, 1991). When children make errors, it is often due to misconceptions of the problem situation grounded in insufficient understanding of the semantic schemes of the word problems. De Corte and Verschaffel (1991, p. 119) in their work found that *word problems that can be solved by the same arithmetic operation but differ with respect to their underlying semantic structure have very different degrees of difficulty.*

Cook (2006) found that college and elementary school students used the same strategies to discriminate between relevant and irrelevant information in word problems. The same strategies were applied also when students did not succeed in solving the word problem. Transition strategies or multiple strategy use were more frequent when students did not manage to solve the problem correctly. Littlefield and Rieser (1993) found that when successful students based their discrimination on a feature analysis of the text, less successful students were more likely to use surface-level aspects like positioning of numbers or number grabbing. Such results were also found by Brekke (1995) in the Norwegian KIM-study, while Cook and Rieser (2005) found that mathematically disabled students attempted to apply the same strategies as successful students, only were not able to implement the strategies effectively. Misinterpretations or surface-based strategies might lead students to automatically consider a word problem an addition problem if they encounter the word *all together* in the text (Cummins et al., 1988; Reed, 1999). Cook (2006) on the other hand suggests that when students do not succeed in constructing a fitting mental model, this could be due to lack of mathematical knowledge (domain knowledge) relevant in the given situation.

The model of domain learning – a framework for describing student competence

The model of domain learning is an alternative perspective on expertise (Alexander, 2003). While research on expertise within cognitive science has given rich profiles of experts and contrasted novice behavior by expert behavior, the model of domain learning describes characteristics of competence at *acclimation*, *competence* and *proficient* levels within academic domains (Alexander, Buehl, Sperl, Fives & Chiu, 2004). A novice student typically

has little and fragmented domain knowledge accompanied with few and rigid strategies and can be described as being at an acclimation level, while an expert will have reached the level of proficiency (Alexander, 1997). The goal of schooling² is to bring students to the level of competence (Alexander, Graham & Harris, 1998).

The model of domain learning is a stage theory, defining competence as consisting of three interrelated parts: domain knowledge, strategic behavior and interest (Alexander, 1997). Differences in domain knowledge and strategy use within the mathematical domain are well documented (see for instance Ostad & Sørensen, 2007 or Schoenfeld, 1992).

Collins, Brown & Newman (1989) claim that the strategic knowledge experts possess usually is tacit knowledge and thereby difficult to verbalise and make explicit and observable to others. Experts have a broad range of strategies that they apply in a flexible manner, allowing them to perform different activities necessary to work on unknown or unfamiliar problems (Alexander, 1997). Often strategies are defined as goal directed and non-obligatory actions (Ostad, 2001). Collins et al. (1989) claim that strategies can consist of both automatic and controlled processes. Within the model of domain learning *strategies are defined as intentional and effortful actions taken when individuals perceive some problem or gap in understanding* (Alexander et al., 2004 p. 547). Alexander et al. (1998 p. 132) stress that *principled knowledge is the hallmark of academic competence, and that strategies are necessary to achieve such principled understanding.*

Researching strategy use

Verbal protocols are traditionally used in research studies where researchers aim at investigating thinking or strategy use. Such protocols in the form of either retrospective or concurrent think aloud protocols, are collections of student talk when conducting an activity (Pressley & Afflerbach, 1995). While retrospective reports are generally considered too reflective to reliably tap into only cognitive and not also meta-cognitive strategies and self reflection, concurrent think aloud protocols are considered appropriate tools for building theory about cognition. The challenges of validity are lesser than with retrospective reports since think aloud protocols are reports of the latest contents of the short term memory (Ericsson & Simon, 1993).

When students think aloud while performing an activity, there is no delay in time between conducting the activity and reporting. Such reports can be termed level 1 reports and

² Primary and secondary school

are considered to be actual reports of the content of working memory and hence strategies in use (Ericsson & Simon, 1993; Pressley & Afflerbach, 1995). However such verbalisations are difficult to children, and they also some times forget to verbalise their thinking. At such occasions the student should be reminded to think aloud (Ericsson & Simon, 1993). If time spans are too long, students might be giving retrospective reports of their reading or solving and thereby expose meta-cognitive understanding or reflections (level 3 protocols) instead of cognitive strategies in use. Still meta-cognitive and cognitive strategies are an equally important part of students' strategy repertoire. Of special interest are strategies in consideration to self monitoring or students' strategies to evaluate answers (Alexander et al., 1998). At the same time it is known from prior studies of higher order thinking skills that using verbal protocols in the form of think aloud protocols to investigate strategies does not affect students' success-rate or test score (Leow & Morgan-Short, 2004; Norris, 1990). For these reasons, both level 1 and level 3 protocols are considered appropriate and have been collected.

Ginsburg, Jacobs & Lopes (1993) suggest that think aloud protocols are a potentially strong tool in order to assess thinking. Verbal protocols applied with scaffolding open up for diagnosing students' zones of proximal development. Scaffolding offered by a more knowledgeable other, adjusts for and is sensitive to the student's difficulties in consideration to the specific word problem. Scaffolding can be described as *controlling those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence* (Wood, Bruner & Ross, 1976 p. 9). Hence a larger portion of students' strategy use is displayed than when left alone and unable to solve the problem. This difference between what students can do when working independently on the word problems and what they can do with scaffolding is what is in a students zone of proximal development (Vygotsky, 1978).

Investigating student strategy use when working on word problems

The main part of the study *Understanding and working on word problems* consists of a protocol analysis. Students have given verbal protocols while working on a collection of word problems. For every student test scores on national tests in numeracy³ have been collected to give individual measures of domain knowledge, while test scores on national tests in reading serve as individual measures for reading comprehension. While the verbal protocols give rich

³ Regning

information about student strategy use, only limited knowledge about students' domain knowledge can be derived from the protocols. While level 1 protocols are appropriate tools to investigate students' strategy use, protocols including scaffolding talks are more appropriate for diagnosing what students can do in their zone of proximal development.

Data collection

22 students⁴ have given verbal protocols while solving eight word problems. Protocols were collected during the last week of teaching in grade seven or during the first fourteen weeks of grade eight. All protocols were given in an interview setting. Students were asked to read and think out aloud while working through the problems as they would during a lesson or if the problem was assigned for homework. The word problems were age appropriate, collected from national tests, exams and text books, some challenging even to very competent students. Both multiple choice and open ended questions were represented. Different levels of complexity were represented. All word problems demanded more than one "step" of calculations in order to be solved correctly. Each problem was printed on top of a separate page (A4), allowing more than sufficient space for taking notes, drawing or performing calculations. Students would work through the word problems in their own pace⁵. They would decide for themselves when a problem was solved and when to turn the page and move on to the next problem. Scaffolding was offered when students got stuck.

Two examples of rereading when working on word problems

In my study to understand is connected to comprehension of word problems: To understand means to create a mental representation of the problem situation to an extent that enables you to solve the problem (see for instance Thevenot et al., 2007; Cook, 2006 or Nathan e al., 1992). A first priority for the overall study will be to investigate how reading and solving strategies work "together". In this paper two examples of how students interact with the text while working on a word problem, are presented and discussed in the light of the suggested framework. The strategy of interest in these two examples is *rereading*. Also students' comprehension of word problems in the form of manifestations of mental representations will be discussed.

⁴ The students come from two different schools situated in a major city in Norway. Both schools are combined primary and lower secondary schools.

⁵ Interviews lasted between 25 and 60 minutes.

The examples presented are both part of level 1 protocols where the student eventually manage to solve the word problem correctly. The analyses of level 1 protocols are based on a priori developed categories, representing findings in other laboratory studies⁶ where the unit of analysis has been students' strategy use. The category *rereading* is applied when students reread a part or all of the word problem text. This category is used both by students who are successful and by students who fail to approach an understanding of the text. As the transcripts demonstrate, *rereading* occur during all phases of working on the problem.

Boy1 – Rereading and elaborating about text elements

In order to arrive at a situation model, students need to comprehend the text of the word problem. If texts are short or easy to comprehend, it is possible to hold the whole text in working memory after reading through once (Pressley and Afflerbach, 1995), but often it is necessary to reread to be able to discriminate between relevant and irrelevant information (Cook, 2006).

Boy1	<i>Anne works in a store after school every Tuesday and Thursday. She makes 80 kroner per hour. When she turns 16, she will get a raise in her wage for five percent an hour. How much will she make an hour when she turns 16?</i>	Initial reading, reads through WP text once
	<i>After school every Tuesday and Thursday. 80 kroner, I think she earns... Five percent. Five percent. How much is five percent more an hour?</i>	Elaborates
	<i>How much will she make an hour when she turns 16?</i>	Rereading
	<i>Five percent more</i>	Identifies relevant information
	<i>Then it is 85.</i>	Suggests solution
I	<i>Mm</i>	
Boy1	<i>I think it will be 85.</i>	Confirms suggestion
I	<i>Mm</i>	
Boy1	<i>I need to find out. I need to. I must try to...</i>	Questions model

Example 1: Boy1WP3 – Anita' wages

Boy1 is a student at acclimation level. His scores on the national tests are well within the score interval for the 20 % weakest students for both reading comprehension and numeracy.

⁶ *Cognitive labs* could serve as a collective term for this research

WP3 is the only one of the eight word problems he solves correctly⁷. One of the reading strategies Boy1 demonstrates for most problems is *rereading*. Rereading can for instance be aimed at discriminating between different text elements or at understanding or elaborating on the content of the text (Pressley and Afflerbach, 1995). In example 1 Boy1 is working on WP3, a word problem that contains both relevant and irrelevant information. Students need strategies to discriminate between these. Many students employ a question orientated strategy to do this (Cook, 2006).

Boy1's elaborations in the transcript is guided towards understanding the problem text, that is creating a mental representation and identifying the relevant information needed to solve the problems as represented. His elaborations could be viewed as either repeating or rewording text elements for elaboration purposes in order to explain parts of the text (Pressley and Afflerbach, 1995). Such elaboration can be viewed as being about the social situation, understanding the context. According to Thevenot et al. (2007) the mental representation is to the main part a more qualitative social model than the mathematical structure of the problem lies within. Boy1's initial structure suggests adding 80 and 5. It is not clear however whether this means 85 kroner or 85 %. Other students adding 80 and 5 demonstrate two different surface strategies: Number grabbing or use of key word. Alexander et al. (2004) labels such strategies as text base strategies. In the Norwegian version of WP3 the term "more" is used in the problem text about the raise in wages. This could suggest that adding is the appropriate mathematical action to students focusing at key words⁸.

Girl1 – Rereading during solving

Both students who succeed in solving a word problem and students who fail choose to reread the problem text. Students who fail to form an appropriate mental representation often reread the whole text several times. According to Cook (2006) these students could either not know how to implement rereading in order to discriminate between relevant and irrelevant information, or they could lack necessary domain knowledge.

Girl1 in example 2, after performing initial calculations, stops to reread the problem text. After finishing solving the problem she explains that the reason for rereading was to check on the numbers in the text – if she remembers them correctly. Other students rereading

⁷ The transcript of his verbal protocol, however, does not show this, only his initial reading and elaborating is shown.

⁸ For use of key words as a surface strategy see for instance Reed (1999).

during solving to check particular information differed in consideration to rereading the whole text or only parts of it. Rereading during solving can be viewed as monitoring. Students reread to check if what they are calculating is an expected answer to the problem question, to adjust their mental representation. Monitoring your own process, to keep on track, is an important part of a student's domain competence (Schoenfeld, 1992; Teong, 2003).

Girl1	<i>Aud bought four T-shirts and three pairs of socks. The T-shirts were 79 kroner each. All together she paid 364 kroner. How much did a pair of socks cost?</i>	Initial reading
Girl1	<i>Ehm. T-shirts are 79 kroner.</i>	Identifies central text element
I	<i>Mm</i>	
Girl1	<i>For one</i>	Relationship
Girl1	<i>And then we multiply with four</i>	Suggests algorithm
I	<i>Mm</i>	
Girl1	<i>Nine times four is nine, eighteen, twenty seven, thirty six. And seven times four is fourteen, twenty eight, twenty nine, thirty, thirty one. And</i>	Performing algorithm Counting
Girl1	<i>(reads in a whispering voice)</i>	Rereading

Example 2: Girl1WP2 – T-shirts and socks

Concluding remarks

Viewing domain knowledge and strategy use as two parts of a whole (students' domain competence), as suggested, seems useful for researching students' strategies for reading and solving word problems. The word problems in the two examples are rather trivial problems to competent grade 8 students. To students at an acclimation level they can however be challenging. To solve them correctly students needs to master appropriate domain knowledge (multiplication, division, subtraction and percentage) as well as reading strategies aiming at comprehending the text of the word problems. Boy1 is a student at acclimation level. Girl1 has slightly higher scores on national tests, scoring just above the 20th percentile. Both solve only one word problem out of eight correctly for the verbal protocols.

According to Alexander's model, to be mathematically competent means different "things" to different people; acclimation students also know something even though it is fragmented and not flexible in respect to domain knowledge and strategy use (Alexander et al., 2004). Even though test scores and protocols suggest that both students might be considered to be at acclimation level, some strategic knowledge as well as some domain

knowledge was demonstrated. Although, unlike in the transcripts⁹, surface comprehension strategies such as number grabbing or wrongful use of key words, were not rare when solving other word problems. The use of text-based strategies rather than deep processing strategies is one characteristic of acclimation competence (ibid.). However use of key word can be a success strategy for some word problems as key words can be strong indication as to the mathematical structure and prototypical character of word problems (Reed, 1999), but then the use of key words is a result of deep processing (Alexander et al., 2004). Also in this case reading strategies at a complex level are paired with domain knowledge at a high level.

References

- Alexander, P. A. (1997). Mapping the multidimensional nature of domain learning. The interplay of cognitive, motivational and strategic forces. *Advances in motivation and achievement*, 10, 213 – 250.
- Alexander, P. A. (2003). The development of expertise: The journey from acclimation to proficiency. *Educational Researcher*, 32(8), 10 – 14.
- Alexander, P.A., Graham, S. & Harris, K. R. (1998). A perspective of strategy research: Progress and prospects. *Educational Psychology Review*, 10(2), 129 – 154.
- Alexander, P. A., Buehl, M. M., Sperl, C. T., Fives, H. & Chiu, S. (2004). Modelling domain learning: profiles from the field of special education. *Journal of educational psychology*, 96(3), 545 – 557.
- Brekke, G. (1995). *Veiledning til tall og tallregning*. Oslo: Nasjonalt læremiddelsenter.
- Bråten, I. (1994). *Skriftspråkets psykologi. Om forholdet mellom lesing og skriving*. Oslo: Høyskoleforlaget.
- Collins, A., Brown, J. S., & Newman, S. (1989). Cognitive apprenticeship: Teaching the crafts of reading, writing, and mathematics. In Resnick (Ed.), *Knowing, learning, and instruction* (pp. 453–494). Hillsdale: Erlbaum.
- Cook, J. L. (2006). College students and algebra story problems; strategies for identifying relevant information. *Reading Psychology*, 27, 95 – 125.
- Cook, J. L. & Rieser, J. J. (2005). Finding the critical facts: Children's visual scan patterns when solving story problems that contain irrelevant information. *Journal of educational psychology*, 97, 224 – 234.
- Cummins, D. D., Kintsch, W., Reusser, K. & Wiemer, R. (1988). The role of understanding in word problems. *Cognitive psychology*, 20, 405 – 437.
- De Corte, E. & Verschaffel, L. (1991). Some factors influencing the solution of addition and subtraction word problems. In Durkin, K and Shire, B (Eds.), *Language in mathematical education. Research and practice* (pp. 117 – 130). Philadelphia: Open University Press.
- Ericsson, K. A. & Simon, H. A. (1993). *Protocol Analysis: Verbal Protocols as data*. Cambridge MA: Harvard University Press.
- Ginsburg, H., Jacobs, S., & Lopez, L. (1993). Assessing mathematical thinking and learning potential. In Davis & Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 237–262). Boston: Allyn and Bacon.
- Hegarty, M., Mayer, R. E. & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of educational psychology*, 87, 18 – 32.
- Leow, R. P. & Morgan-Short, K. (2004). To think aloud or not think aloud. The issue of reactivity in SLA research methodology. *Studies in second language acquisition*, 26 (1), 35 – 57.
- Littlefield, J. & Rieser, J. J. (1993). Semantic features of similarity and childrens' strategies for identification of relevant information in mathematical story problems. *Cognition and Instruction*, 11, 133 – 188.
- Nathan, M. J., Kintsch, W. & Young, E. (1992). A theory of algebra word problem comprehension and its implications for the design of learning environments. *Cognition and Instruction*, 9, 329 – 389.
- Norris, S. P. (1990) Effect on eliciting verbal reports of thinking on critical thinking test performance. *Journal of Educational Measurement*, 27 (1), 41 – 58.
- Nortvedt, G. A. (unpublished work). *Grade 7 students' strategies for reading word problems*.
- Ostad, S. A. (2001). Matematikkvansker i et longitudinelt perspektiv. In Dahlvang, T. Formo, Lunde, O. og Bekken, O. (Eds.), *En matematikk for alle i en skole for alle. Rapport fra det 1. nordiske forskerseminar og matematikk vansker*. Kristiansand: Info Vest forlag.

⁹ The transcripts are transcripts of parts of problems they solve successful.

- Ostad, S. & Sørensen, P. M. (2007). Private speech and strategy-use patterns: Bidirectional comparisons of children with and without mathematical difficulties in a developmental perspective. *Journal of learning disabilities* 40 (1), 2 – 14.
- Pressley, M. & Afflerbach, P. (1995). *Verbal protocols of Reading*. Hillsdale, NJ: Lawrence Erlbaum.
- Reed, S. K. (1999). *Word problems. Research and curriculum reform*. Mahwah, NJ: Lawrence Erlbaum.
- Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In Grouws, D. (Ed.), *Handbook of research on mathematic teaching and learning* (pp. 334 – 370). New York: MacMillan.
- Semadeni, Z. (1995). Developing children's understanding of verbal arithmetical problems. In M. Hejný & j. Novotná (Eds.), *Proceedings of the International Symposium on elementary Math Teaching* (pp. 27 – 32). Prague, The Czech republic; Faculty of Education, Charles University.
- Teong, S. K. (2003). The effect of metacognitive training and mathematical word problem solving. *Journal of computer assisted learning*, 19, 46 – 55.
- Thevenot, C., Devidal, M., Barrouillet, P & Fayol, M. (2007). Why does placing the question before an arithmetic word problem improve performance? A situation. *Quarterly journal of experimental psychology*, 60, 43 - 56.
- Verschaffel, L., Greer, B. & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.
- Vygotsky, L. (1987). *Mind in society*. Cambridge, Massachusetts: Harvard university press.
- Wood, D., Bruner, J. & Ross, G. (1976). The role of tutoring and problem solving. *Journal of child psychology and psychiatry*, 17, 89 – 100.