

Mathematics: Grounded Enlightenment - or Pastoral Salvation
Mathematics, a Natural Science for All - or a Humboldt Mystification for the Elite
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This mini-paper is a condensed version of the original rejected paper asking what insight might come from a postmodern perspective on mathematical knowledge. Using anti-pastoral sophist research identifies two forms of schooling, US enlightenment schooling and EU Humboldt counter-enlightenment schooling, together with two different forms of mathematics, enlightenment mathematics and pastoral mathematics.

Background. Pre-modern Enlightenment mathematics presented mathematics as a natural science. Exploring the natural fact multiplicity, it established its definitions as abstractions from examples, and validated its statements by testing deductions on examples. Inspired by the invention of the set-concept, modern mathematics turned Enlightenment mathematics upside down to become 'metamatics' that by defining its concepts as examples of abstractions, and proving its statements as deductions from meta-physical axioms, needs no outside world and becomes entirely self-referring.

However, a self-referring mathematics soon turned out to be an impossible dream. With his paradox on the set of sets not being a member of itself, Russell proved that using sets implies self-reference and self-contradiction known from the classical liar-paradox 'this statement is false' being false when true and true when false: 'Definition: $M = \{A \mid A \notin A\}$. Statement: $M \in M \Leftrightarrow M \notin M$ '.

Likewise, without using self-reference it is impossible to prove that a proof is a proof; a proof must be defined. And Gödel soon showed that theories couldn't be proven consistent since they will always contain statements that can neither be proved nor disproved.

With the failure of set-based metamatics you wonder why school mathematics has not replaced it with Enlightenment mathematics. Maybe a postmodern perspective can give an answer.

Anti-Pastoral Sophist Research. Ancient Greece saw a fierce controversy between two different forms of knowledge represented by the sophists and the philosophers. The sophists warned that to protect democracy people needed to be enlightened to tell choice from nature to prevent the emergence of patronization presenting its choices as nature. The philosophers argued that patronization a the natural order since everything physical is an example of a meta-physical form only visible to the philosophers educated at Plato's academy, who then naturally should become the patronising rulers.

The Greek democracy and the sophist warning vanished with the Greek silver bringing wealth by financing trade with Far-East luxury goods as silk and spice. Later the sophist warning came back in the Enlightenment period: when Newton has shown that an apple follows its own will, people could do the same and replace patronization with democracy. Two democracies were installed, one in US, and one in France. US still has its first republic, France now has its fifth.

The German autocracy tried to stop the French democracy by sending in an army of mercenaries that was no match to the French army of highly motivated conscripts ending up occupying Germany. Unable to use the army, the German autocracy instead used the schools asking Humboldt to create an elite school to stop the spread of the enlightenment schools from France. Humboldt replaced grounded enlightenment with Spirit-based Bildung and created the Humboldt university protected by a Humboldt gymnasium only allowing the best half to enter, and only allowing the best half to go on to the Humboldt university only allowing the best half to graduate in one of its many office-educations, and still present in the EU.

Inside the EU the sophist warning is kept alive only in France in the postmodern thinking of Derrida, Lyotard and Foucault warning against pastoral patronising categories, discourses and institutions presenting their choices as nature (Tarp 2004). Derrida recommends that pastoral categories be 'deconstructed'. Lyotard recommends the use of postmodern 'paralogy' research to invent alternatives to pastoral discourses. And Foucault uses the term 'pastoral power' to warn against institutions legitimising their patronisation with reference to categories and discourses based upon choices claimed to be nature.

Anti-pastoral sophist research can't refer to existing research; instead this is deconstructed by asking 'In patronization, what is nature and what is pastoral choice presented as nature?' To make categories, discourses and institutions not pastoral but enlightening, they should be grounded in not choice but nature, e.g. by using Grounded Theory (Glaser 1967), the method of natural research developed in the other Enlightenment democracy, the American; and resonating with Piaget's principles of natural learning (Piaget 1970) and with the Enlightenment principles for research: Observe, abstract categories and relations to be accommodated through predicted deductions.

The Nature of Numbers. Feeling the pulse of the heart on the throat shows that repetition in time is a natural fact; and adding one stick and one stroke per repetition creates physical and written multiplicity in space. A collection or total of e.g. eight sticks can be treated in different ways. They can be rearranged to an eight-icon containing the eight sticks, written as 8. They can be collected to one eight-bundle, written as 1 8s or $1 \cdot 8$. And they can be ‘decimal-counted’ by bundling & stacking, bundling the sticks in e.g. 5s and stacking the 5-bundles in a left bundle-cup, and stacking the unbundled singles in a right single-cup. When writing down the counting-result, ‘cup-writing’ gradually leads to decimal-writing where the decimal point separates the bundle-number from the single-number: $8 = 1 \text{ 5s} + 3 \text{ 1s} = 1)3) = 1.3 \text{ 5s}$.

The Nature of Operations. Operations are icons describing the process of counting by bundling & stacking. The division-icon ‘/2’ means ‘take away 2s’, i.e. a written report of the physical activity of taking away 2s when counting in 2s, e.g. $8/2 = 4$. The multiplication-icon ‘4*’ means ‘(stacked) 4 times’, i.e. a written report of the physical activity of stacking 2-bundles 4 times, $T = 4 \cdot 2$. Subtraction ‘- 2’ means ‘take away 2’, i.e. a written report of the physical activity of taking away the bundles to see what rests as unbundled singles, e.g. $R = 9 - 4 \cdot 2$. And addition ‘+2’ means ‘plus 2’, i.e. a written report of the physical activity of adding 2 singles to the stack of bundles either as singles or as a new stack of 1s making the original stack a stock of e.g. $T = 2 \cdot 5 + 3 \cdot 1 = 2.3 \text{ 5s}$. Thus the full process of ‘re-counting’ or ‘re-bundling’ 8 1s in 5s can be described by a ‘recount or rebundle formula’ containing three operations, together with a ‘rest formula’ finding the rest: $T = (8/5) \cdot 5 = 1 \cdot 5 + 3 \cdot 1 = 1.3 \cdot 5 = 1.3 \text{ 5s} = 1.3 \text{ bundles}$, since the rest is $R = 8 - 1 \cdot 5 = 3$.

The Choices of the Humboldt Curriculum. Primary school introduces the ‘natural’ numbers one by one using the follower-principle. Thus ten is introduced as the follower of nine, which quickly introduces 2digit numbers and place values. Later comes multi-digit numbers. Likewise addition is introduced first to practise earlier numbers adding up to the actual number. Then subtraction is introduced as taking away and counting up to. Multiplication and the tables follow; and in the end division and simple fractions.

Middle school enlarges the number domain with fractions, decimals and percentages to be treated with all four operations. Proportionality is introduced as well as equations and early algebra with letter calculation and linear functions. In geometry the focus is on 2- and 3-dimensional forms.

High school enlarges the number domain with irrational numbers, and the operations are enlarged with power, roots and logs. In equations algebraic letter fractions are included, and different forms of functions are introduced: polynomials, exponential, circular. Often a short introduction to calculus takes place. In geometry the focus is on coordinate geometry, trigonometry and some vector geometry.

A Natural Enlightenment Curriculum. In primary school multiplicity leads to counting and re-counting by bundling and stacking reported by cup-writing and decimal-writing, predicted by the recount formula $T = (T/b) \cdot b$. The operations are introduced in the order /, *, -, +. 1digit mathematics includes equations and calculus. 2digit numbers are introduced via overloads thus avoiding the cognitive bomb ten, the only number with a name but without an icon.

In middle school the focus is on per-numbers. Proportionality is the recount formula used for double-counting a quantity in two different units. Adding fractions and percentages includes the units as in integration. Modelling leads to linear formulas. Formulas with two unknowns are graphed and with one unknown solved using reversed calculations, first reducing a multiple calculation to a single by placing the hidden parentheses, and then moving numbers to the other side reversing their calculation signs. Geometry is earth-measuring leading to right-angled triangles seen as a rectangle halved by a diagonal, and where the height and length can be recounted in diagonals giving sine, cosine and tangent as percentages.

In high school adding constant per-numbers leads to exponential change. Adding variable per-numbers generalises middle school’s adding fractions with units to finding the area under the per-number graph. Reversed addition then leads to roots and logarithms and to differentiation. Coordinate-geometry is included.

Conclusion. A postmodern perspective on mathematics shows that in spite of its failure modern set-based mathematics is kept in schools because it serves well the goal of the Humboldt counter-enlightenment schooling practising segregation to sort out the elite. In primary school it introduces ten as a natural number where decimal stack-numbers are the natural numbers, thus installing ten as a cognitive bomb. In middle school it introduces mathematism when insisting that fractions be added without units. And in high school it introduces metamatics when insisting that functions be the core concept and that calculus be based upon the limit-concept. So today’s school mathematics is not nature, but a choice becoming pastoral by suppressing its natural alternative enlightening the nature of multiplicity by using decimal counting in primary school, by adding fractions with units in middle school, and by making high school function and limit-free.

References. The full paper will be placed on www.MATHeCADEMY.net