

DIFFERENT VERSIONS OF THE 'SAME' TASK: CONTINUOUS BEING AND DISCRETE ACTION

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Introduction

In this paper I probe subtle differences in lessons which are based around similar tasks, by analysing the experiences they afford students, and identifying what is available for students to construct from these experiences. This provides a new lens for looking at mathematical activity in lessons, and at how teachers' own mathematical senses act out to afford different mathematical experiences for learners.

There is a resurgence of interest in task design as an important factor in mathematics teaching (Sierpiska, 2004; Burkhardt and Schoenfeld, 2003). An international society has been founded and a number of publications show that design has to be taken seriously not only for extended, multi-stage, authentic and assessment tasks (as described by, for example, Wittman 1998), but also for the very ordinary things we ask students to do day-to-day in classrooms (among others: Swan, 2006 ; Mason and Johnston-Wilder, 2006 ; Karp, 2007). Runesson (1999) Emanuelsson (2001) and their colleagues offer mathematical variation as a critical feature of the process of task design while others (e.g. Watson 2004) focus on the affordances of tasks as structures for potential mathematical activity. Variation informs us about affordances.

However, over-reliance on task design as a vehicle for improvement in mathematics teaching is known to be a flawed approach on its own. There is a steady history of research which shows how the designers' intentions become altered as their tasks are taken up and used in classrooms when teachers apply their own perspectives and local purposes (Stein, Grover and Henningsen, 1996; Stylianides and Stylianides, forthcoming; Palhares et al, forthcoming). Emphasis on the ensuing activity – what is actually done, talked about, learnt, and how this takes place – places pedagogy and culture alongside tasks as equally important factors in the resultant mathematical qualities of the classroom.

Analysing lessons

For about ten years I have been developing descriptions of the mathematical and pedagogical choices which make a difference to learners' mathematical experiences. Various attempts to do

this have been published, and yet whenever I think a particular approach is ‘finished’ another arises from my reflections on classroom observations, and my own mathematical activity, alone or with others. Analysis which starts with questions and prompts (Watson & Mason, 1998; Watson, 2007) focuses on what learners are asked to do; analysis which starts with examining variation among presented examples (Watson and Mason, 2006) focuses on the material from which learners can generalise. Both of these are different from analysis which starts from comparing mathematical possibilities offered through the teacher’s role in interactive sequences (Watson, 2004; David & Watson, 2007); this latter approach to analysis tends to focus on how learners’ engagement can be mathematically shaped. There are other methods of analysis around, such as categorising different epistemological aspects of mathematics which are emphasised during a lesson (e.g. Andrews & Sayers, 2005). All these approaches are valuable, but none completely capture the full sense of how one lesson is mathematically different from another. Mathematical activity appears to be fractal, and to unfold differently depending on the starting focus (see also Davis & Sumara, 2006). By unfolding layers of activity from one perspective, the folds hide other aspects which might be equally important.

The nature of mathematical activity

For this paper I have returned to mathematics itself, and its assumed structures, concepts and definitions, to think about differences in lessons. In the TIMSS seven-nation video study ‘mathematical quality’ was ‘measured’ by a team of well-qualified mathematicians (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs, et al. 2003) and used as a comparative characteristic. The categorisations were very vague, it being assumed that people with string mathematical qualifications can make such judgements. Something more informative is needed for teachers and teacher educators whose interpretations of ‘mathematical quality’ are necessarily limited by their own mathematical experiences. Variation theory (Marton, Runesson and Tsui, 2004) provides a tool for doing this to some extent, but I am going to show that there is more. I do this by looking at classroom activity associated with ‘the same’ task taught by different teachers.

Rather than looking at tasks to predict activity (using the distinction developed by Christiansen & Walter, 1986), I am going to look at the nature of public mathematical activity to find a new lens for seeing task implementation.

My method is to use classroom observation and video to reflect on the nature of classroom mathematics. To focus on ‘public activity’ means to ask the questions: ‘What is the class supposed to be doing right now? What are they supposed to be thinking about? What is being said and done, and by whom, that is shaping and is shaped by the activity?’

Using the ‘same’ tasks

Several teachers in the same school were teaching groups of 12 year-olds who were more or less similar in previous attainment.¹ All teachers agreed to use a similar approach to teaching loci using a combination of straight-edge-and-compass constructions and the physical whole class activity of acting out loci by following instructions to ‘find a place to stand so that ...’ (e.g. ‘find a place to stand so that you are the same distance from these two points’; or ‘... all two metres from this point’; etc.) All classes constructed circles, perpendicular bisectors of line segments, and angle bisectors and some other loci. An indoor open space was available to do the physical task, and teachers chose to do this at different points during their lessons. Students were intended to relate their physical experience of standing according to such rules to the processes of geometrical construction. This kind of connection, enabling shifts between three very different representations (words, actions and diagrams), is one of the characteristics noticed by Krutetskii as typical in gifted mathematics students (1967). The problem for teachers is, therefore: how can all students be helped to make the connections that the highest achieving students are expected to make for themselves? This problem is exacerbated by the affordances of the physical task: it is possible to take a ‘gap-filling’ role without constructing a personal interpretation of the instructions, and hence not to have an experience of being a point in relation to other points to refer to when reproducing the locus on paper. It is also worth mentioning that these students had little experience of geometry beyond some knowledge of angles, and the naming of polygons – no classical, formal, geometry at all.

The five lessons were compared qualitatively in a variety of ways. I looked at the nature and amount of variation offered in the task, the questions and prompts used by teachers, whether teachers worked with whole classes or small groups, interaction patterns, combinations of ‘doing’

¹ The data on which this analysis is based was collected by my colleague Els De Geest during work on a joint project funded by Esmee Fairbairn Foundation (05-1638); the analysis is my own responsibility and the school context is disguised.

and ‘thinking’ prompts, the emphasis on reasoning, and whether teachers simplified their questioning when students could not, at first, answer. These foci for analysis were selected on an underlying theory that students can only respond to what is made available to them in the words, actions and artefacts of the lesson. In other words, the mediational devices and instructions used by the teacher and other students, whether intentional or not, shape the learners’ experience of the lesson. In these five lessons, this shaping turned out to be mathematically different even though the actions and artefacts were similar. The details of the data are omitted here in order to focus more quickly on what I claim to be essential differences, these having been arrived at by comparing features of lessons under the categories described above.

There were strong similarities between the lessons: all teachers used a mixture of asking, prompting, telling, showing, referring students to other students’ work and so on. All teachers focused on getting students to explain their choices and actions. All students had to work out as much as they could themselves about how to do the constructions, either by reasoning or by listening to others’ reasons in whole class discussion. The tasks were presented in remarkably similar ways and, in variation theory terms, offered similar variation in similar ways due to the mathematical structures being taught and the choice of loci with which to work, which had been agreed by the team. Teachers’ intentions were similar, and all of them praised accuracy and sought for reasoned action. Written work was similar, and students might report similar experiences after the lessons. Analyses in terms of variation and affordances and constraints, and situational norms, and the nature of questions and prompts, and the kinds of demands made on learners provided very similar results. None of the dichotomies used in the literature to compare lessons superficially (open/closed; teacher-centred/learner-centred; traditional/reform) were helpful in identifying difference, yet as a mathematical observer I know that the mathematical affordances of the lessons varied. They provided different kinds of intellectual and mathematical engagement.

The components of the tasks were offered in different orders by different teachers; teachers said different things to students at different times; there was a range of different patterns of participation for individual students in each lesson; the various possible constructions were offered in different orders.

To express these different kinds of engagement I shall now describe three different versions of this lesson and draw attention to different mathematical learning experiences for the students involved. To do this, I am drawing on the five observed lessons, constructing *compendium* lesson outlines which include the main common features, described above, of the five; in fact one lesson is as it was originally, while the other two are each built from features of lessons which had a great deal in common. In this way I present lessons which have in common many of the features usually highlighted in current literature about lesson analysis, yet this presentation allows me to also present differences of intellectual and mathematical affordances. I am using this approach for several reasons:

- It is tempting when offered lesson descriptions to compare to make judgements about overall quality, based on beliefs about superficial features of ‘good’ lessons. Constructing compendium lessons avoids this tendency.
- Compendium lessons conceal the identities of teachers who may feel that their lessons are being compared unfairly, without full knowledge of previous and subsequent lessons
- It provides a way to include significant contrasts succinctly

As a research method this is a valid procedure because everything that is included is from an actual lesson, and hence has authenticity and credibility in the field. Further, I am going to use the compendium lessons to claim is that there is a way of looking at teaching that transcends detailed lesson descriptions. Compendium lessons are fit for this purpose because my final claim does not rest in individual details.

Lesson one

The lesson started with students working as a class, with guidance from the teacher, working out how to use a pair of compasses to construct circles, a locus with constant distance from a straight-line segment, perpendicular bisectors and angle bisectors. The teacher repeatedly referred to compasses as the tool for reproducing equal lengths: he said this himself, and also asked students ‘what can we use to get equal lengths?’ and ‘what do compasses do for us?’ and ‘why would I use the compasses?’ Students were then asked to compare the perpendicular bisector and angle bisector constructions, and to identify the role of compasses within these. The words ‘same distance’ and ‘equidistant’ were used frequently throughout the lesson. The teacher invited

students to demonstrate their ideas on the board, and also used the strategy of placing ‘wrong’ points to encourage students to understand the role of constraints. The physical activity took place at the end of the lesson, and was treated briefly as a summary of the rest of the lesson. Technical language was used throughout.

The focus on the power of the tool was reinforced by comparing its role in constructing the two different bisectors, so that students were looking at the positions of, and relationships between, the equal lengths in the constructions. By taking this approach, learners were able to talk about relationships within the diagrams as if they were caused by the equal lengths, rather than equal lengths merely being a drawing method. It was made possible for them, by this focus, to get a sense of classical geometrical tradition. The physical activity at the end used the same language of ‘equal lengths’ in instructions and descriptively where necessary and offered no further public engagement of mathematical thinking, merely rehearsal in a different context, using a different representation, with nothing said about how to make equal lengths in physical action.

Lesson two

In this lesson, students were asked to locate points which fulfilled certain rules: points which are all the same distance from another point, two points, two lines and so on. This was done on the whiteboard with discussion, and also on paper, the initial approach being consisting of rough diagram and reasoning, and compasses being introduced later as a way of joining up the points for the circle and locating particular points. The emphasis was on how to use them, rather than why they worked. The predominant language was about points which ‘obey rules’. The word ‘locus’ was introduced during discussion of where all such points would be. During the second part of the lesson students took part in the physical representation of loci in response to the same language as was used to find points. It appeared that the students were expected to link the different parts of the lesson (pencil and paper construction, whiteboard drawing and physical activity) through the use of the same language to express the same ‘rules’ for placing points and people, and an increasing use of the word ‘locus’. One student called out: ‘this is what we have just done!’

In each case the emphasis was on collections of points, each of which has a particular property, and on joining up the points. The role of the compasses was not emphasised; they were treated as

a means to join up points which are equidistant from other points. Verbal instructions about finding individual points with properties were the most repeated sound of the lesson.

Lesson three

In the third lesson, the physical activity took place first, and the teacher offered a story to encourage visualisation: standing the same distance from two trees; steering a ship between two icebergs. Students then returned to the classroom and were asked to construct the same loci as had been acted out physically.

The physical activity happened first so that students were expected to have some memory to draw on when they came to make constructions in pencil and paper. No public instructions for constructing were given, instead students were asked to work out how to do them using their memories. The teacher worked hard with small groups of students asking them what they remembered and how they could reproduce it. In general she said ‘you can use the compasses’ when equal lengths were needed, sometimes showing them how to do it and then asking them to do it again for themselves. A significant amount of time was given at the end of the lesson to the task of developing statements that linked the physical activity to the pencil-and-paper constructions. Students had to express the isomorphisms between the situations.

Discussion

The three lessons were likely to have left different traces in students’ minds about what the key ideas were about loci:

- trajectories derived from relationships between equal lengths;
- sets of points which have certain properties;
- reproductive constructions of physical situations.

From a mathematical viewpoint these are equivalent, but in terms of learning experience they are different and memory of the lesson content is likely to be triggered by different stimuli in future. I am reluctant to arrange these in any sort of hierarchy of mathematical challenge: each invites learners to shift from obvious, intuitive visual and physical responses to the more formal, ‘scientific’, responses required for mathematics. In each of these lessons there are emphases on relationships between variables, properties, reasoning about properties and relationships among properties, so available hierarchies based on assumptions about cognitive challenge and ways of

seeing (e.g. van Hiele 1959) do not identify difference – and yet different mathematics is learnt – or at least the ‘same’ mathematics seen, described, and triggered in different ways. It is important to sustain the delicacy of these differences in mathematical terms, rather than to dive into pedagogic differences between the lessons (e.g. how much groupwork, what sort of questions, patterns of interaction etc.) which will give more information about some aspects of the lessons but less information about mathematical didactic structure.

Task differences

In each of the lessons above, interpretations of the task have been made by individual teachers, after team planning. In these lessons we do not see any reduction of challenge as is reported about adoption of published tasks. The teachers have discussed common approaches, which loci should be used, and how lesson should be resourced in terms of space and equipment. Overt activity is similar; a more casual observer might say they were the same lesson. The effects of tool use were the same, although the sense of appropriation might be different, and the mathematical content was equivalent.

What differed was what was emphasised by the teacher, but I am not saying that this was merely talk. Rather, the difference was, I claim, due to the underlying general relationships within which the teacher saw the task as being embedded. Because teachers see these differently they therefore use different language, different sequencing and different emphases so that different comparisons and connections can be made – yet all of these are equally mathematical.

In each lesson such differences were continuous. Each lesson was coherent throughout in the relationships among its tasks, language, emphases, prompts and other components. Each lesson was an expression of how the teachers saw the links between the tasks, tools and learners within their understanding of what loci generally entailed.

This realisation releases me from attempts to describe good mathematics teaching as a collection of actions, utterances, tasks, and examples, and instead leads me to look for the continuous threads of mathematical awareness the teacher is revealing by her/his actions and decisions. We can then see teaching mathematics as the more-or-less fluent expression of an understanding of a mathematical context for the current work.

The implications of this insight are that we can see mathematics teaching as a way of being mathematical, and the education of mathematics teachers as a mathematical experience.

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