

Mathematics in the Upper Secondary Electricity Program in Sweden: A Study of Teacher Knowledge

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Mathematics teachers' and electricity teachers' explanation of a mathematical task in the electricity context has in this study been compared and characterized. By studying the teacher knowledge in mathematics and in electricity that the teachers draw upon in explaining this task, some similarities and differences have been found. Detailed descriptions of specific explanations of the same topic by two teachers have been studied and it will further contribute to a suggestion of how the teachers' arguments and reasoning can be characterized.

Introduction

Mathematics is used in many vocational courses in the Swedish upper secondary education and mathematical knowledge is often a prerequisite to learn an occupation. At least one mathematics course is mandatory for students enrolled in the Swedish gymnasium and according to the Swedish national curriculum documents the teaching of the mathematics course should be adapted to the specific program that the students are studying at (Skolverket, 2000). The practical use of mathematics could be visible to the students, by using material and experience from the vocational program (Lindberg & Grevholm, 2011). But it is not clear that the linking the mathematics course to the specific program that the students are studying at really do enrich the vocational courses and the mathematics course.

For example at the Electricity and Energy program, the national curriculum documents states that to be able to do correct calculations is a prerequisite in order to exercise the profession and the education should therefore develop the students mathematical knowledge (Regeringskansliet, 2011). The students at the Electricity and Energy program use mathematics in two different courses: their mathematics course and their electricity course. The students are taught by two different teachers, the mathematics teacher respectively the electricity teacher, and those teachers have different education and background. Some mathematics is needed to do electricity work, but what does that mathematics look like at the secondary school level? Students are actually getting two very different kinds of treatments of mathematics by their two different kinds of teachers (Straesser, 2007). But what, exactly, is this difference? That is what this article will start to

examine. In this study mathematics and electricity teachers are interviewed about what explanations they would give to students to a mathematical task in the electricity context, a task commonly used in the first electricity course and a task that could be used in the mathematical course at the Electricity program. The task presented in this article involves calculating the total resistance in a parallel electrical circuit with two given parallel resistances.

The aim of this study is to explore the similarities and differences in mathematics and electricity teachers' explanations of mathematical electricity tasks to start understand how mathematics in the electricity context looks like when presented to students at the Electricity program.

What teacher knowledge in mathematics and electricity do mathematics and electricity teachers at the Electricity program use in explaining a mathematical electricity task?

Are there characteristic similarities and differences in the teacher knowledge that the teachers draw upon to explain these electricity tasks?

Theory

Teacher knowledge is a widely researched area. 1986 Shulman introduced pedagogical content knowledge (PCK) as an important part of teacher knowledge, besides the subject matter knowledge (SMK) and general pedagogical knowledge domains (L. S. Shulman, 1986). To be able to compare the teachers in this study, the teachers' PCK and SMK in both mathematics and in electricity are studied and Shulman's definitions are used. SMK refers to the amount and organization of the subject in the teachers' minds and goes beyond knowledge of the facts and concepts in order to understand the subject's structures. PCK refers to explanations and representations of the subject that make the subject understandable to learners, and further it includes knowledge of what makes the learning of the subject easy or difficult and what preconceptions learners are likely to have and also knowledge of strategies that are likely to help those learners.

Teacher knowledge have been used in a wide range of studies in mathematics education (Hill, Rowan, & Ball, 2005; Loewenberg Ball, Thames, & Phelps, 2008; Rowland & Ruthven, 2011) and in studies comparing different mathematics teachers (Krauss et al., 2008; Ma, 1999). Teacher knowledge has also been used in science education (Loughran, Mulhall, & Berry, 2004; Magnusson, Krajcik, & Borko, 1999) where electricity is included as a topic. Studies of teacher knowledge in the vocational education literature are rare.

Method

In this study I have interviewed 8 mathematics and electricity teachers. The teachers have been given a mathematical task within the electricity context and they have been asked how they would explain and help students on the electricity program with this task. This method is used to understand the nature and extent of teacher knowledge (Hill, Sleep, Lewis, & Loewenberg Ball, 2007) and has

been used in studies comparing different mathematics teacher groups (Ma, 1999). The tasks used are examples commonly used in the first electricity course at the electricity program (Niss & Højgaard Jensen, 2002) and the teachers' written explanations have been videotaped.

The group of teachers consists of three mathematics teachers and five electricity teachers, who are teaching in the first year at the Electricity program at four different schools in northern Sweden. The questions in the interview tasks were on the form of how to explain this task to a student who does not understand the task and they were told to give alternative explanations when they could. The video camera was directed downwards and captured the paper where the teachers wrote and pointed at the electricity circuits in the tasks. The interviews, that lasted approximately one hour each, were transcribed.

Analysis of data

What the teachers say and do (calculate, point to, draw pictures and so on) is regarded as indications of knowledge that they have and draw upon to explain the interview task. The teachers' statements and actions are used as indications of their teacher knowledge, where "something will count as knowledge, and be modeled as knowledge, if it appears to be used as such by the individual being modeled" (Schoenfeld, 2011).

A summary of the teachers' statements and actions during each interview task was made. This summary consists of identified portions of explanations, expressing one idea each (Marks, 1990) and one teacher's explanations of a task consists of a list of several different portions of explanations.

In this study Shulman's definitions of teacher knowledge are used (Shulman, 1986). Subject matter knowledge (SMK) is defined as knowledge of the content, both conceptual and procedural knowledge. Pedagogical content knowledge (PCK) is defined as knowledge of useful representations, examples and illustrations to make the content accessible to the students. Also included in PCK is knowledge of common students' conceptions, misconceptions and difficulties with the content.

The analysis started with the teachers' summaries; where each portion of explanations in the summaries was coded as indications of mathematical knowledge or electricity knowledge, depending on the main idea of the statement. For example a teacher's explanation of the formula for the total resistance in a parallel electrical circuit, with a simpler number example, and calculations of this example with both decimal numbers and fractions, this was categorized as indications of mathematical knowledge. A teacher's explanation of the same parallel circuit task, talking about electrons moving in the circuit, this was categorized as indications of electricity knowledge. After the first categorization in mathematics or electricity knowledge, all the indications of knowledge in the mathematical category were divided into SMK or PCK and the same was done with the electricity knowledge category. Teachers' statements and actions of facts, concepts and structures of the subject are categorized as SMK. Explanations and representations of the subject and also knowledge of

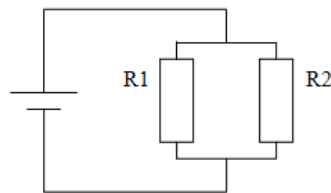
students' preconceptions are categorized as PCK. For example teachers' calculation of the total resistance in a parallel circuit was categorized as SMK in mathematics and a teacher's explanation with a simpler number example to help a student understand the formula is categorized as PCK in mathematics.

These categories of SMK and PCK in mathematics and electricity were studied to explore the similarities and differences between the two teacher groups.

Results

This section gives an overview of all the teachers' explanations of a interview task, see Figure 1, and after that a presentation of detailed analyses of one specific explanation given by one mathematical teacher and one electricity teacher are given.

Your students have started to work with electrical circuits with parallel resistance. Many students have difficulties in using the formula to calculate the total resistance in a parallel circuit.



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

R1 is 220 ohm and
R2 is 120 ohm

Task:

Calculate the total resistance in the circuit.

How would you explain how to solve this task to a student?

Figure 1. Interview task given to the teachers during the interview.

An overview of the teachers' explanations of this task is presented in a picture showing the teachers' explanations in four rows representing SMK and PCK in mathematics and SMK and PCK in electricity, see Figure 2.

Overview analysis

The teachers were in the interviews asked to present all possible ways of explaining the task to a student (Figure 1). The overview picture (Figure 2) shows all explanations presented by all teachers, and this overview shows that the teachers use explanations that are similar, but there are several explanations that one of the teacher groups gives and not the other teacher group. This overview shows that in SMK in mathematics there is one explanation exclusively used by mathematics teachers and the same for electricity SMK, where the electricity teachers have explanations exclusively used by electricity teachers. Interestingly, electricity teachers have some indications of knowledge in the category PCK in mathematics that the mathematics teachers do not use, and the mathematics teachers have one indication of knowledge in PCK in electricity that the electricity teachers do not use.

Indications of Teacher knowledge, task 2

Subject matter knowledge in mathematics

Say that rounding too early gives a wrong answer M3	Calculating the total resistance in the parallel circuit M1-3, E1-5
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Pedagogical content knowledge in mathematics

Explain that the task is too difficult for students to solve with fractions M1, M3	Show easier examples with fractions in the beginning M1, M3, E2, E3	Explain how to get R when you have $1/R$ M1, M3, E2	Explain that to use a structure in the solving of a task makes it easier for the students E2, E3	Say that some students use to forget to invert in the end of their calculations E2, E3, E5	Say that students can have a bad self-confidence in mathematics E1
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Subject matter knowledge in electricity

Say that the total resistance in a parallel loop is always less than the smallest resistance M1, M3, E2, E3, E4, E5	Say that if the resistances are equal then there is an easier formula E3, E4, E5	Calculate an estimation to know what answer is plausible E5
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Pedagogical content knowledge in electricity

Explain the circuit diagram by talking of electrons that are moving in the conductors M1, M3	Say that the two new concepts, serie and parallel circuit, may be difficult for the students M1, E1, E2	Use a model to explain the circuit e.g a concrete example with bridges over a river or traffic on roads M1, M3, E1, E3, E5	Explain with reasoning that the current will be bigger with two conductors, so the total resistance must be less M1, M3, E1, E4, E5	Explain the origin of the formula: $1/R=1/R1+1/R2$ M3, E4
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Figure 2. Yellow boxes show explanations given only by mathematics teachers; blue boxes show explanations given only by electricity teachers and green boxes shows explanations given by both mathematics and electricity teachers.

M1-M3, represents mathematics teachers, E1-E5 represents electricity teachers.

This is somewhat surprising, given that the mathematical teachers are educated in mathematics education and the electricity teachers are educated in electricity education. That teachers of one group have unique PCK in the other teacher group's speciality could be an indication of that pedagogical content knowledge is something that is developed with experience of teaching (L. Shulman, 1987).

Detailed analysis

The overview (Figure 2) leaves out details that were found interesting during the interviews and transcriptions. These details do not show up in this first analysis, so a new analysis section began, where explanations were selected that in this overview were categorized in the same category, but at a closer look had differences. For example has Marks (1990) highlighted the fact that the level of analyze in teacher knowledge is important.

In this article, a detailed analysis of one mathematics and one electricity teachers' specific explanations of the same topic have been done, and a

suggestion of how to characterize the differences between the teachers' explanations is made. The explanation that is analysed is the green box farthest to the right in the bottom row in the overview, Figure 2, and it shows one mathematics teacher's and one electricity teacher's explanation of the origin of the formula for the total resistance in a parallel circuit.

The detailed study of one mathematics and one electricity teachers' explanations indicates several differences in the teacher knowledge that the teachers draw upon when arguing for and reasoning in their explanation. This finding suggests that an explanation can be supported by different knowledge of different types. The teachers' arguments and reasoning for his/her explanation can be supported by both mathematical knowledge and by electricity knowledge. Both the mathematical and the electricity knowledge could be of either specific or general type. In this analysis, an explanation based on general mathematics consists of a general algebraic solution, and an explanation based on specific mathematical knowledge consists of a solution with specific numbers for the specific task. An explanation based on general electricity knowledge consists of theoretical electricity arguments and reasoning. An explanation based on specific electricity consists of a real world illustration or a concrete example to illustrate this task, see Table 1. One teacher's explanation could consist of several arguments and reasoning based on both mathematical and electricity knowledge of both general and specific types. In the next section, one mathematics teacher's explanation and one electricity teacher's specific explanations of the same topic, will illustrate how these constructs of general and specific mathematical knowledge, and general and specific electricity knowledge could be used in an analysis to highlight the differences in teachers' explanations.

Knowledge/Type	Specific	General
Mathematical knowledge	Concrete numbers	Algebra
Electrical knowledge	Real world illustration/ concrete example	Theoretical/Physics

Table 1. Definitions for the detailed analysis.

Detailed analysis data

The two teachers explain the origin of the formula for the total resistance in a parallel circuit. Both teachers start with Kirchhoff's current law ("Kirchhoff's laws," 2009) stating that the sum of the part currents in the parallel branches equals the head current. After that the mathematics teacher used an algebraic way to show how the formula origin from Kirchhoff's law. The electricity teacher on the other hand, showed the origin of the formula by reasoning and calculating in the electricity circuit.

Mathematics teacher*Teacher wrote:*

$$I = I_1 + I_2$$

$$U = I \cdot R$$

$$I = \frac{U}{R}$$

$$\frac{U}{R_{ers}} = \frac{U}{R_1} + \frac{U}{R_2}$$

$$U \cdot \frac{1}{R_{ers}} = U \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$U \cdot \frac{1}{R_{ers}} / U = U \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) / U$$

$$\frac{1}{R_{ers}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Teacher did:

The teacher started with writing Kirchhoff's law, and explained it while pointing to the part currents and head current in the circuit diagram.

He said that the voltage is the same over the two resistances and that the currents could be rewritten by Ohm's law.

The teacher substituted the currents in Kirchhoff's law with expressions for the currents derived from Ohm's law.

The teacher then factorized the expressions on both sides of the equal sign,

and he divided by U on both sides of the equal sign.

He cancelled U/U and got the formula for the total resistance in a parallel circuit.

In this way the teacher proved the origin of the formula for the total resistance in a parallel circuit, by using algebra. The teacher used electricity knowledge in the beginning when he used Kirchhoff's law and when he explained that the voltage is the same over the two resistances and the same as the voltage source. This is categorized as general electricity knowledge, since it is broad knowledge about electrical circuits. Thereafter he used mathematical knowledge characterized as general mathematical knowledge, merging two formulas and simplifying the merged formula, which is not specific to only this task.

Knowledge base	Type	The teacher's explanation
Mathematical	General	Uses algebra to rewrite Ohm's law for the part currents
Mathematical	General	Merge Kirchhoff's law with Ohm's law for the part and the head current
Mathematical	General	Breaks out U from the expressions on both sides of the equal sign in the Kirchhoff's law equation and cancels those U
Electrical	General	Starts with Kirchhoff's law, $I=I_1+I_2$
Electrical	General	Reasoning that it is the same U over R1, R2 and the voltage source

Electricity teacher

The electricity teacher said that he in class use to show the formulas origin by calculating the total resistance in the circuit with the aid of different chosen voltages.

Teacher wrote:

$$I = I_1 + I_2$$

$$U = 9V$$

$$I_1 = \frac{9}{220} A = 0.041A$$

$$I_2 = \frac{9}{120} A = 0.075A$$

$$I = I_1 + I_2 = 0.041 + 0.075 \\ = 0.116A$$

$$R_{\text{ers}} = \frac{U}{I} \Omega = \frac{9}{0.116} \Omega = 77\Omega$$

$$\frac{120}{R_{\text{ers}}} = \frac{120}{R_1} + \frac{120}{R_2}$$

$$\frac{1}{R_{\text{ers}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Teacher did:

The teacher started with Kirchhoff's law, and said that the sum of the part currents in the circuit diagram equals the head current.

He showed in the circuit diagram that it is the same voltage over the both resistances. He made an experiment choosing the voltage 9 V in the circuit

and calculated the part current I_1 using the voltage 9V, and the part current I_2 also using the voltage 9V.

The teacher calculated the head current as the sum of the part currents.

The total resistance was calculated with Ohm's law and the voltage and the head current in the circuit.

After that the teacher said that he uses to try other voltages, and he gets the same total resistance independent of what voltage he chooses, for example 12V or 120V.

The teacher said that in the given formula the voltage is 1 V. He said that as long as it is the same voltage as numerator in the formula the total resistance will be the same. He also said that the advantage with having the voltage 1V is that you have the button on the calculator (1/x), which makes it easy to calculate this.

The mathematical knowledge this teacher used is categorized as specific mathematical knowledge, since he is calculating the electrical magnitudes (currents and resistance) in this particular circuit. Also the electricity knowledge the teacher used is categorized as specific electricity knowledge, when he chose different voltages to use in his calculations. The teacher concluded that the total resistance in the circuit is independent of what voltage there is based on his calculations and this was categorized as specific mathematical knowledge, as this conclusion is built of specific number examples. The teacher also used general electricity knowledge when he introduced Kirchhoff's law and explained that the voltage is the same over the two resistances and the same as the voltage source, since this is broad knowledge about electrical circuits.

Knowledge base	Type	The teacher's explanation
Mathematical	Specific	Calculating the part and head currents in the circuit for different chosen voltages
Electrical	Specific	Chooses a voltage to be able to calculate currents and calculates part and head currents and total resistance in the loop
Mathematical	Specific	Makes the conclusion that the total resistance in a parallel circuit is independent of what voltage there is, based on the concrete examples that have been calculated
Electrical	General	Starts with Kirchhoff's law, $I=I_1+I_2$
Electrical	General	Reasoning that it is the same U over R1, R2 and the voltage source

Comparison of the teachers' explanations

Both teachers' explanations were based on their knowledge of Kirchhoff's current law in a parallel circuits, general electrical knowledge of electrical circuits. But the teachers used different argumentation and reasoning in the continuation of their explanations. The mathematics teacher used general mathematical knowledge and the electricity teacher used specific mathematical and electrical knowledge.

Conclusions and discussion

This interview study of mathematics and electricity teachers' explanations of a mathematical task in the electricity context shows both similarities and differences between the two teacher groups' explanations, but as only eight teachers were interviewed this gives only a glint of how this could look like at schools. At the overview level, with the teachers' explanations categorized in subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in both mathematics and in electricity, it appears that the two teacher groups' explanations are mostly similar, but this overview could not show all the details of the explanations. A detailed analysis of a specific explanation of a single topic suggests that a teacher's explanation is based on argumentations and reasoning that could be characterized in general and specific mathematics and electricity knowledge, but further studies are needed to refine this.

The detailed analysis showed that the mathematics teacher used general mathematics, whereas the electricity teacher did not. Other studies have shown that general mathematics is not part of workplace mathematics which may be one reason also here. "From a mathematical point of view, efficiency is usually associated with a general method that can then be flexibly applied to a wide variety of problems.....The crucial point is that orientations such as generalizability and abstraction away from the workplace are not part of the mathematics with which practitioners work." (Noss, Hoyles, & Pozzi, 2000). The electricity teacher in this study used specific mathematical and specific electricity knowledge, sufficient for his explanation and also discussed in workplace mathematics research: "For the practitioners in our studies, the computational and estimation methods of routine activity – in all there many forms – were more than adequate for their purposes." (Noss, et al., 2000).

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