The Emergence of Mathematical Meaning and Disciplined Improvisation

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This paper presents a theoretical argument for the construct of disciplined improvisation as a style of pedagogy that promotes the emergence of mathematical meaning. I argue that problem solving provides one context for disciplined improvisation with a balance of structure and openness. This paper illustrates this by reporting on an example from a lesson in a primary (elementary) school that demonstrates the mathematics that can emerge when conditions are set up for disciplined improvisation. I suggest that explicitly introducing teachers to the discipline behind disciplined improvisation may be a way of providing pedagogic strategies that teachers can use to structure lessons for emergence of meaning.

Introduction

Matusov (1998) distinguishes between two views of development – as coming about either through internalisation or through participation – and argues that a preference for one or other of these positions comes from different world-views. In discussing Matusov's work, Daniels (2001) distinguishes between 'skills and functions' in the 'internalisation thesis' and 'meaning' in the 'participation thesis'. The effects of these world-views or theses can be detected in the tensions and contradictions within which mathematics teachers find themselves positioned. On the one hand there is increasing pressure from centrally determined curricula and increased regimes of testing, ostensibly designed to ensure that students achieve particular learning outcomes. Much advice to teachers on ensuring such outcomes can be located in the internalisation view of development and a skills and functions view of teaching whereby effective teaching techniques can be identified, specified and implemented. This can also be thought of as predicated on a view of teaching (and learning) operating as a complicated system (Davis and Sumara 2006) and, like other complicated systems such as analogue clocks, can thus be 'managed' and 'adjusted' to bring about pre-determined outcomes.

On the other hand there are calls to teachers to encourage learners' creativity and to help them develop 'productive dispositions' that go beyond regarding learning mathematics as merely mastering a collection of techniques. Such 'soft' learning outcomes cannot be engineered into being in the same way that specific skills (apparently) can. With advice to teachers couched in terms of 'facilitation' and 'affordances', teaching for creativity and productive dispositions is more rooted in the participation thesis and a view of classrooms and schools as complex systems. In contrast to the high predictability of complicated systems, in complex systems the precise outcomes of events cannot be known in advance. Just as the analogue clock is the archetypal metaphor for a complicated system, so gardens are typical of complex systems. Cutting down, say, a bush in a garden will bring about change, but whether that change will result in the system flourishing or deteriorating cannot be known in advance. Changes in a complex system are not predictable, but neither are they random and changes are, theoretically, explainable after the event. In contrast to the language of mechanics and engineering that characterises complicated systems, complex systems are described in terms of organics and emergence.

So while theorists (and some teachers) might argue for the recognition of teaching and learning operating as a participatory complex system, policy directives, in the main, continue to take a internalisation position, positioning teachers as 'deliverers' of mathematical knowledge that, given appropriate instruction, should not be problematic for learners to acquire. How then are teachers to reconcile being caught between these two positions? In this paper I use Sawyer's (2004) construct of 'disciplined improvisation' as a metaphor that I suggest is helpful in reconciling these oppositions.

Theoretical background

Collaborative emergence

Sawyer (2001) traces the origins of the concept of emergence to work in 1875 by the philosopher George Henry Lewes and Lewes' distinction between two types of effects: resultants and emergents. The main qualities of emergent effects, Sawyer argues, are that outcomes cannot be fully understood or predicted by studying the constituent parts, as illustrated by Lewes' example of the effect of water emerging from the combination of oxygen and hydrogen. Understanding the properties of water cannot fully be achieved by reduction to the study of the properties of oxygen and hydrogen (although quantum physics now overturns this claim). This non-reductionist aspect of emergent phenomena means that they are multiplicative rather then additive in their nature (Davis and Simmt 2003). Although the concept of emergence has been developed since Lewes' time, particularly in the physical sciences, it probably began to have most impact on educational research with the development of artificial intelligence systems that displayed intelligent behaviour based on simple, local rules of interaction and without the need for a central leader. Thus models of how insect colonies create complex structures is a canonical example of an emergent system (Clark 1997). Theorising from such simple forms of emergence it has become generally accepted that group behaviour can be considered as emergent when there is no structured plan for the group to follow, and where there is no leader directing the group (Sawyer 1999). Classrooms and students are, however, fundamentally different from anthills and ants in the range of actions and agency available to the participants. To distinguish between systems where there is interaction but not agency, in the sense that individuals within the system can intentionally change the direction of what is emerging, I use Sawyer’s phrase of collaborative emergence to encompass phenomena “that result from the collective activity of social groups” (1999, p 449).

Many social activities might be considered to be examples of collaborative emergence, for example, football matches or jazz performances. An analogy I find helpful is that of theatrical improvisation (‘improv’), which when applied to teaching and learning, Sawyer suggests is best described as ‘disciplined improvisation’ (Sawyer 2004).

Disciplined improvisation

> It’s a common misconception about improvisation that performers simply play whatever pops into their heads, that “anything goes.” Improvisation, although it involves spontaneity and extemporizing, doesn’t mean that there is a total lack of structure. (Sawyer 2000 p 180)

The everyday use of the word ‘improvise’ carries connotations of the audience’s perspective of being unstructured and completely spontaneous. In particular, talk of a lesson being improvised smacks of a teacher not being prepared and simply ‘winging it’. From, however, the players’ perspective theatrical improvisation is inherently disciplined. Although to the audience it may appear that an improvised scene is simply the result of actors ‘freewheeling,' from the performers’ perspectives what emerges is grounded in a structure of rules and principles. Although from the insider (that is player’s) position the idea of ‘disciplined improvisation’ may seem a tautology, the phrase is helpful in drawing attention to that fact that to promote improvisation as an aspect of teaching is not to advocate a lack of preparation or acceptance that anything that arises in a lesson is fine.

Improvisation in classrooms must also be disciplined in the sense of the discipline of knowing when enough is enough — when to accept and build on learners’ offerings and when to direct the lesson back in a particular direction. A challenge is to plan lessons that are sufficiently open to allow for the possibility of learners making contributions that can be built upon but at the same time sufficiently structured to allow for the mathematics that emerges to be to be worthwhile.

I argue that problem solving provides one approach to planning for a balance of structure and openness, for disciplined improvisation. Research has shown that teachers often try to ‘over-structure’ problem solving lessons and effectively close down the opportunities for meaning to emerge.

This paper illustrates this by reporting on an example from a lesson in a primary (elementary) school that demonstrates the power of the mathematics that might emerge when attention is paid to learners’ offerings and these are built upon. The example comes from a class of nine-year-olds struggling to understand fractions and illustrates how improvised solutions to a problem can form the basis of emergent meaning.

I suggest that explicitly introducing teachers to the discipline behind disciplined improvisation may be a way of helping them appreciate the potential within open problem solving and also provide a set of pedagogic strategies that teachers can use to structure lessons without closing them down.

The school context

This example comes from a two-year teaching experience in a primary school, Bow Bells, in the East End of London. The school is located in a traditionally working-class area that more recently has attracted a high immigrant population. At the time of this teaching measures of performance judged by national tests, only around 45% of pupils at age eleven were attaining the expected level in the tests, compared to government targets of 80%. Inspection reports painted a picture of a school in difficulty, which did not attract teachers to apply to work there. The school was thus in a downward spiral and to counteract this the local authority had put in a new head-teacher, a specialist in literacy.

At that time I was looking to go back to do some school teaching. Several years working in research had revealed little evidence of the sort of problem-solving based pedagogies that are written about and I had begun to wonder if teachers were right in sometimes claiming that academics were wrong and that, given the constraints of schooling, problem-solving based teaching was not possible. In approaching the local authority for a school to work in, Bow Bells was suggested.

At initial meetings with the teachers two things were frequently commented upon. First, they talked about the limited language facility of the children (even for those children for whom English was their first language) and that consequently there was little point in asking the children to talk about mathematics. Second, and linked to the first point, there was a general sense that the children had little to contribute to mathematics lessons: it was important to equip children with the ‘basics’ before they would be able to engage in any form of problem solving or reasoning. This attention to the ‘basics’ permeated throughout the school from the classes of five-year-olds to the eleven-year-olds and the predominant style of teaching across all the years was one of the teachers demonstrating a method on the board and the children subsequently completing practice worksheets.

A colleague, Penny Latham, and I agreed with the staff that rather than accept these perceived limitations as givens our main focus would be on supporting the children in being able to talk about mathematics and to develop their mathematical understanding from problems and problem solving.

I have set out this context at some length as I want to make it clear that the children we were working with were not 'privileged' or had had the kinds of experiences that might pre-dispose them to problem-based pedagogies. The example that follows is typical of the sort of work we did. It comes from a class of eight- and nine-year olds, towards the middle of a year of work with them and their teachers. The lesson was co-taught by the regular class teacher and myself.

Example: Sharing pizzas

The lesson began with a whole class discussion setting up the context for the problem — going out with friends or family for a pizza. We did not rush to introduce the actual problem, taking time to set up an atmosphere in the classroom that we hoped conveyed the spirit that 'we', the teachers, were interested in what offerings 'they', the children, would bring to the problem.

Finally we posed the problem, orally:

> 12 friends went out for a pizza. It was towards the end of the month, so they only had enough money to order 8 pizzas. They ordered the 8 and shared them equally. How much pizza did each person get?

The children cooperated in small groups, recording workings and solutions on one large piece of paper with a single marker pen. While they worked on the problem we (the teacher and I) went round encouraging groups to show and explain their methods. As answers began to emerge we selected two groups to share their solutions with the class and told these children to be prepared to come up to do this. The first method (Figure 1) was typical of what many groups had done: everyone got half a pizza each and the remaining two pizzas could be sliced into sixths. So the friends each got 1/2 + 1/6 pizza.

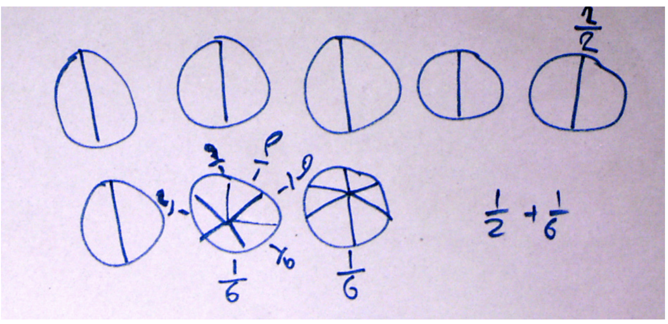

Figure 1: Everyone gets 1/2 + 1/6

The other solution was based on sharing four pizzas so that everyone would get 1/3 pizza (Figure 2). The second four pizzas would provide another 1/3 each. So in this group each person would get 2/3 of a pizza. (Figure 2 shows that some work needed to done later on in helping the children represent thirds as equal sized pieces, but the principle of their argument was correct.)

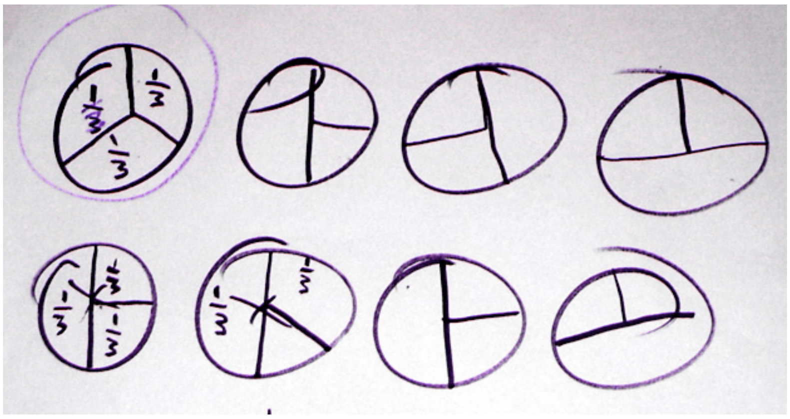

Figure 2: Everyone gets 2/3

After both solutions had been presented and discussed, the class was in agreement that each was correct. But what was happening here? Going out with one group could mean getting 1/2 + 1/6 of a pizza to eat; going with the other group could give you 2/3 pizza. Were these the same? If you really liked pizza which group would you want to go out with?

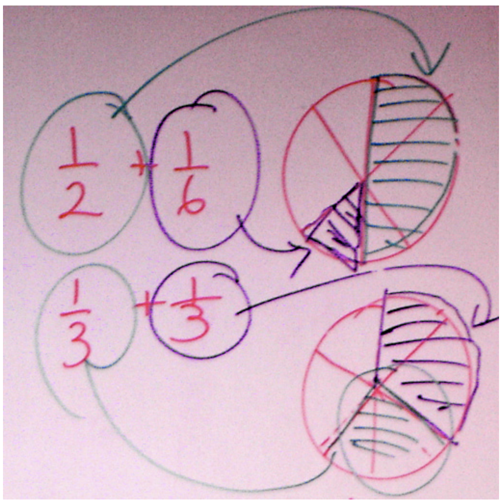

Figure 3: Everyone gets the same

This is a point at which the interplay between the improvised solutions and the formal mathematics come into play. The children knew from their everyday knowledge that, given the sharing was fair in each case, the portions would have to be equal. The challenge was to sort out why, mathematically, these two solutions appeared to be different. Back in their groups we challenged the children to come up with some representation that would show whether the two amounts were equal or different. Figure 3 is typical of the diagrams they produced to show equality. The plenary discussion about what they had learnt indicated that many of the children were beginning to get a sense of what equivalence was about

A pedagogy of disciplined improvisation.

It is easy to suggest that this example is merely a manifestation of a move from a teacher-centred to a learner-centred pedagogy, but I want to argue that such labelling diminishes both the work of the teacher and the children. I do this by linking Davis and Simmt's theoretical conditions for emergence to the practices of disciplined improvisation and the pizza problem as an example of disciplined improvisation in the classroom.

Drawing on complexity theory (e.g., Johnson, 2001) Davis and Simmt identify five conditions that they consider necessary (but not necessarily sufficient) for the emergence of collective meaning:

- internal diversity
- internal redundancy
- decentralised control
- enabling constraints
- neighbour interactions.

Internal diversity is the result of variations among the group and is necessary for a range of possibilities to emerge. Internal diversity is a given in improv, indeed it is what drives the improvisation forward. As players pick up each other's offerings these get changed through the diversity. Take, for instance, the miming of offering a gift. Although in the mind of the player making the offering it might be clear that the gift is a necklace, the player accepting it could announce anything from an anklet to a zebra skin or any of an infinite number of possibilities.

In the pizza example the class was of heterogeneous ability and the learners grouped into mixed attainment groups. The discussion in these small groups and the posters the learners created to represent their different approaches to solving the problem were outcomes of this diversity. From this perspective of disciplined improvisation, diversity in classrooms is desirable and an asset rather than something that teachers need to 'manage' (through differentiated activities) or 'reduce' (through ability grouping practices).

Internal redundancy is a complement to internal diversity. Redundancy in Davis and Simmt's sense means a degree of sameness in the form of "commonalities of experience, expectation, and purpose" (p. 310). Internal diversity promotes variety, while internal redundancy both facilitates the participants' interactions and allows for gaps in any one participant's contributions to be compensated for by the others. Internal redundancy is built into improvisation both through the offers that are not taken up, and also in the way that, as a scene emerges, the range of possibilities is reduced and players work within the parameters of what is then allowable and sensible. In the pizza problem the choice of context built in internal redundancy. The children had enough common knowledge of pizzas to be able to build on each other's offering. For example no one produced a representation of a pizza that could not be made sense of by everyone in the group and everybody had a good understanding of 'fair shares'.

Decentralised control is based on the view of complex systems as self-organising. Although the behaviour of a collective may look as through there is some central coordination, this sense of wholeness is actually emerges from locate interactions (Varela, 1999). The dominance of the 'if … then' thinking arising from complicated systems may be a cultural origin of assuming that events are centrally determined through cause-and-effect rather then emerging through complexity (Johnson, 2001). Decentralised control is a core tenet of improv (unlike scripted drama with the control of the director). As soon as one player tries to 'hog' a scene or another 'wimps out' the piece falls apart. Good scenes emerge from sharing the control. In the pizza problem control over the methods of solution was decentralised with the teachers learning from the students and using this to shape the plenary discussion rather than delivering a scripted explanation of equivalent fractions. I agree with Davis and Simmt's observation that it is not helpful to think of such activity in terns of being teacher-centred or learner-centred, mainly because 'the phenomenon at the 'center' was not an individual, but the collective phenomenon of a shared insight (p. 311).

I suggest that a key practice for establishing decentralised control in classrooms is spending time getting the learners to 'buy into' the problems. In improv, the opening minutes of a scene are spent establishing it; from the interplay of offers and take-ups players establish who the characters are, their relationship to each other and where the scene is taking place. Establishing this shared context is similar to what Becker (2000) in his analysis of jazz improvisation calls "a real shared interest in getting the job done" (p. 175). Like other researches leading to rich pupil solutions (for example Fosnot and Dolk 2001) the time spent at the beginning of the lesson setting the context for the problem was not simply window-dressing or a device to make some unpalatable calculations acceptable. There was a general 'suspension of disbelief' created by spending time setting up the scenario, in getting buy-in from the children. This is in contrast to some views that artificial problems do more harm than good. While I would agree that the quick word problem about shopping, followed by another about cooking does not encourage engagement, I think more use could be made of more extended narrative scenarios to hook children in

Enabling constraints permit decentralised control and enable the emergence of phenomena: “Complex systems are rule-bound, but those rules determine only the boundaries of activity, not the limits of possibility. (Davis and Simmt, 2006, p. 311). In improv enabling constraints set boundaries that, somewhat paradoxically, enable to the emergence of scenes that are more than the sum of the offerings of individual players. Key rules here are ‘yes and …’ — accepting and building upon the offers of others — and not ‘pre-scripting’ the scene — mentally writing how you want it to go in advance of its emergence. In the pizza problem the children were organised into groups of three or four and the problem to work on carefully structured, but within that organisation there was diversity in the methods and approaches to finding solutions.

Neighbour interactions, for Davis and Simmt, have to be considered as operating beyond the literal interpretation of learners working together. “Rather, the neighbors that must ‘bump’ against one another are ideas, hunches, queries, and other manners of representation” (p. 312). All improv scenes are built upon neighbour interactions and players words and actions — the offerings - ‘bump’ against each other. In the classroom neighbour interactions were occasioned at two levels: the interactions at the group level and the subsequent interactions between the groups.

One key feature arising from such bumping together of ideas is that the meaning attributed to an offering may only become apparent after the event, of ‘reverse causality’ (Matusov, 1998, p. 330).

Experienced improvisers testify to reverse causation. Although at the beginning of a scene, improving actors have a whole range of options open to them (indeed, one of the disciplines of improv is to keep these options open for as long as possible), once the form and content of the scene starts to emerge, actors will talk afterwards of the scene ‘writing itself’.

The group that solved the problem by sharing the pizzas out in thirds did not set out with that intention in mind: it just happened that the size of paper and their choice of how big to draw the pizzas resulted in them producing a representation of two rows of four. Only having produced this representation did the possibility of sharing in thirds become apparent. The meaning emerging from the representation of two rows of four was ‘distributed across time, space, and participants, interpreted and renegotiated’ (Matusov, ibid., p. 330). Meaning thus emerged from participation in the improvisation of a solution rather than this being a manifestation of some pre-existing understanding that the learners had already acquired and were deliberately bringing into being in solving the problem

Conclusion

Disciplined improvisation provides a structure for looking at and working with Davis and Simmt's suggestion that we need to move attention away from what must happen in lessons to being open to possibilities, from a perspective of prescription to proscription. Proscription, Davis and Simmt argue, is more about setting out what is forbidden in contrast to a prescriptive stance of what is allowed (and by implication everything else is forbidden). In the case of the problem worked on here, the children were not allowed to work alone, and the expectation had been set up that they needed to show on paper how they were setting about solving the problem. There was no expectation that the problems be solved in a particular way (or even that a satisfactory solution be reached at all). Thus a range of possibilities was opened up for meaning to emerge. Explicitly working with teachers on improv is one way of introducing them to the possibilities arising from proscription rather than prescription.

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