Intentions with and Interpretations of the Concept of Problem

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This study examines the connection between intentions with and interpretations of the national curricula documents in mathematics. The study tries to clarify how the concept of problem is intended to be interpreted in the national curricula documents and compare with how the teachers actually interpret it. The results show that that the national curricula documents do not explicitly define what a problem is and that the teachers' interpretations are very varying.

Introduction

In different mathematics education research articles, the concept of mathematical problem is defined and/or used very differently: "(...) problem solving has been used with multiple meanings that range from 'working rote exercises' to 'doing mathematics as a professional'" (Schoenfeld, 1992, p.2). Similarly, Stigler and Hiebert (2004) notes that the "word 'problem' clearly means different things to different people" (p.13). This diversity regarding the concept is reflected in everyday language, at least in both Swedish and English. For example, one English online dictionary defines problem as 'a source of perplexity, distress, or vexation' or 'a proposition in mathematics or physics stating something to be done' (merriam-webster.com). At the same time, the concept of problem is included in the national curricula documents for mathematics in many countries around the world, for example Denmark, England, China, and Sweden[1]. In Sweden, the national curricula documents are the legal documents that are ment to guide the teachers' work. The documents are written in a very compressed way and concepts like problem, problem solving, and reasoning are used to formulate learning goals but are not defined. The use of undefined concepts with many possible meanings gives rise to concrete difficulties for the teachers when it comes to using the curricula documents as guiding tools. The purpose of this study is therefore to deepen the understanding of the connection between intentions with and interpretations of the national curricula documents in mathematics.

This article is part of a large overarching project aiming at clarifying the role of national standardized tests in educational reforms, through the study of the introduction of competency goals in the Swedish national curricula documents (more about this competence reform in the Background). Part of the data collection was carried out in cooperation with the Swedish Schools Inspectorate (see

Bergqvist, Bergqvist, Boesen, Helenius, Lithner, Palm, & Palmberg, 2010) where more than 200 teachers were observed and interviewed on many topics, for example how they interpret the national tests and the national curricula documents in mathematics. Another part of the project is focused on analyzing the Swedish national curricula documents that specify the goals for mathematics education.

In this particular article we examine the concept of problem for many reasons. Firstly, it is a fairly common word in the national curricula documents and it is neither defined nor exemplified. Secondly, it is also a common word in everyday language, as well as in research literature and teacher education, where it has many different meanings. Thirdly, during the above mentioned observations of lessons, episodes that contained problem solving related activities contained higher presence of activities related to other competences (e.g. communication or reasoning), and we see it therefore as an especially central concept. Using data already collected in collaboration with the Swedish Schools Inspectorate, we wish to further clarify how teachers interpret the national curricula documents.

Background

The view on mathematics education is affected by opinions concerning for example philosophy, politics, and education, and therefore standards change over time (Niss, 1991). In the middle of the 1990s, the Swedish government launched new curricula for both compulsory (Skolverket, 1994a) and upper secondary school (Skolverket, 1994b). Both documents were valid from fall 1994 to spring 2011. The documents describe curricula goals "less in terms of subject matter and more in terms of concepts, context, and in knowledge as instrument for learning" (Lundgren, 1999, p.39). In this article we will call this increased emphasis on process-oriented goals a *competence reform*. The national curricula documents in mathematics for upper secondary school were part of this change, which was inspired by several NCTM documents (The Swedish National Agency for Education, 1997) and many references to competence goals can be found (Palm, Eriksson, Bergqvist, Hellström, & Häggström, 2004). The NCTM Standards (2000) is one international example of a well-founded and research-based reform framework describing process or competency goals. There are several other frameworks that build on the idea to complement content goals with competency goals; e.g. the KOM-project (Niss & Jensen, 2002). In several of these frameworks, *problem solving* is one of the included competencies.

The competence reform changed the demands on the teachers (Lundgren, 1999; Remillard, 2005) but the relation between policy and practice is complicated. Change of state standards, the formal curriculum, does not necessarily imply change in student learning, since the teachers' intended and/or enacted curricula are influenced by for example teachers' experiences and beliefs (Stein, Remillard, & Smith, 2007). The cognitive change of teachers can be difficult to achieve

and there are many factors that influences the teachers' readiness to accommodate to new ideas (Gregoire Gill, Ashton, & Algina, 2004). Furthermore, how a text, or even a particular word, is interpreted by the reader is influenced not only by the context of the word, but also by the discourse and community of meaning available to the reader (Hill, 2001). When teachers do not have access to the intentions or discourse of the policymakers they might interpret the concepts through a filter of their own experiences and preferences (Hill, 2001; Bergqvist et al., 2010). In the particular case of the Swedish competence reform, but also in the international counterpart, words like reasoning, problem, and communication are central in the description of mathematical activity. No concepts are however defined or explained in the Swedish national curricula documents. How teachers use and understand such key concepts is therefore an important part of research in mathematics education.

Purpose and research questions

The purpose of this study is to deepen the understanding of the connection between intentions with and interpretations of the national curricula documents in mathematics. Research questions:

1. What meaning of *problem* is intended in the Swedish national curricula documents in mathematics for upper secondary school (course Mathematics A)?
2. How do Swedish upper secondary school teachers interpret *problem* as it is used in the national curricula documents in mathematics?

Method

To answer the first research question the context of each occurrence of the word *problem* (together with its conjugations) in the Swedish national curricula documents for course Mathematics A in upper secondary school (Skolverket, 2000) is examined. Official documents commenting on the national curricula documents in mathematics (Skolverket, 1997) are also analyzed. Any sentence or paragraph containing the word *problem* is searched for expressions describing *what a problem or what problem solving is*.

The data used to answer the second research question comes from interviews during the collaborative work with the Swedish Schools Inspectorate. The sample of teachers is quite representative since it was semi-randomized and carried out by the Swedish Schools Inspectorate. The principle was to include schools from the whole of Sweden, rural as well as urban districts, small as well as large schools, and high-performing as well as low-performing schools. The study includes 53 schools from upper secondary level. In the interviews the teachers were asked questions on many issues related to goals for their own teaching and the students' learning of mathematics. The interviews were quite long (1-1,5 hours) and concerned issues that many teachers expressed as difficult. The data consists

of answers from 126 upper secondary mathematics teachers to one particular question. In one part of the interviews the teachers were presented excerpts from the national curricula documents and one quote concerned the concept of problem. The excerpt came from the grading criteria and was "Pupils use appropriate mathematical concepts, methods, models and procedures to formulate and solve different types of problems" (Skolverket, 2000). The teachers were asked "How do you interpret this wording?" and "How do you interpret the word problem?" The teachers' answers to the last question are analyzed in this paper. The teachers were not asked how they interpret "problem solving" during the interview so there is no data at this time on that issue.

The answers are analyzed using a framework containing categories based both on definitions in dictionaries and definitions relevant in mathematics education. The rationale for combining these two sources is that since the national curricula docuements do not define the concept, the teachers have to make their own interpretation. It is reasonable that they do so either by using general definitions from everyday language (represented by dictionaries), or that they are influenced by teacher education or in-service training courses (represented by definitions relevant in mathematics education). The framework therefore contains a rich sample of definitions of problem in six Swedish dictionaries/encyclopedias: i) The Swedish Academy's dictionary (abbr. SAOB), ii) The Swedish Academy's vocabulary (SAOL), iii) The Swedish National encyclopedia online (NE), iv) The Swedish Wikipedia (W), v) The peoples' lexicon (PL), and vi) Synonymer.se (SY), a public list of synonyms. The framework considers two possible definitions of problem relevant for mathematics education research and school mathematics: A) any task and B) a task for which the solution is not known in advance. Definition A includes any kind of mathematical tasks, for example in textbooks and tests. Definition B denotes a certain type of mathematical tasks that is a subset of the group of A-tasks. Both A and B are definitions that make sense in a mathematics classroom (which might not be the case for all possible meanings found in the dictionaries). The first definition, A, is important since it is sometimes suggested in literature (Schoenfeld, 1992), it is the widest possible interpretation of the word in the context of school mathematics, and if a teacher uses this particular interpretation of the word problem and the intent is something more specific, e.g. B above or a difficult task, then the communication between the curricula documents and the teacher is truly deficient. The second definition, B, is chosen because it is the definition in the NCTM Standards, which was one important inspiration for the present national curricula documents in mathematics and because it is also used as definition of the concept in the overarching project.

The framework has the following categories of definitions of problem: 1. task to solve, version A above, 2a) question: something that one is ignorant of,

version B above, 2b) question: issue, matter, 3a) difficulty: trouble, 3b) difficulty: challenge, 3c) difficulty: riddle.

The analysis was carried out in two steps. First one researcher analyzed all 126 answers. In this process four additional categories were formulated (more detailed descriptions of these categories can be found in the results section): *a task set in a context, word tasks, circular definitions,* and *other*. In the second step the second researcher analyzed all 126 answers using the ten categories from the first step. The results of the analyses show that the two researchers agreed on 103 out of the 126 answers, which indicates a reasonable inter-judge reliability.

Results

The concept of *problem* in the national curricula documents

The first research question is: *What meaning of* problem *is intended in the Swedish national curricula documents in mathematics for upper secondary school (course Mathematics A)?* In the documents (including grading criteria) for Mathematics A (the first mathematics course in upper secondary school) the word *problem* is used 26 times in different contexts (Skolverket, 2000). There are no explicit definitions of the concepts of *problem* (or *problem solving*). For the concept of *problem* there are some pronounced properties presented in the text, e.g. that problems can be solved, analyzed, and critically judged, that different methods can be used to solve problems, and that there are new types of problem of more complex character. In general, however, there is no explicit formulation that specifically separates *problems* from *any tasks*.

For *problem solving* there are fewer occurrences, and most of them do not concern what problem solving is. There is however one passages in the text that explicitly states that problem solving does not mean solving any task: "mathematical problem solving is a creative activity" (Skolverket, 2000). In connection to this quote, problem solving is also stated to be a time-consuming activity, something that further separates problem solving from solving any task.

In an official publication commenting on the compulsory school national curricula documents in mathematics the development of problem solving is described as a central purpose of all mathematics education (Skolverket, 1997). There is no similar official document commenting the national curricula documents for upper secondary school, but in articles and websites documenting the work that preceded the (then) new national curricula documents (see e.g. ncm.gu.se/node/3737), national curricula documents for both compulsory and upper secondary school are mentioned and discussed. We therefore conclude that both documents are based on the same intentions and definitions of the words used. Therefore we use the text in the comments on the compulsory school national curricula documents for mathematics to support our interpretation of the national curricula documents for upper secondary school.

In the comments, the concept *problem* is not explicitly defined but the term *genuine problem* is used in the following context:

> The textbooks' tasks have since the early textbooks in arithmetic been dominated by pre-formulated problems with exactly the numbers to use in the solution given in the test text. Sometimes it is not even a genuine problem since the needed calculation method is given through the context or the chapter heading, so that the students paradoxically enough can solve them without taking the intended everyday context into consideration (Skolverket, 1997, p.18, authors' translation).

We interpret the concept of a *genuine problem* as something different than just any task, for example a routine textbook task, and therefore the concept genuine problem does not belong to category 1a. The intended meaning of the concept is more likely similar to category 2a) *something that one is ignorant of* or 3) *difficulty*, since the solution method is not given according to the quote. There is no direct evidence that the word *problem* in this context means anything else that *any task*, especially since the terms *pre-formulated problems* and genuine problems are used. However, the heading for this paragraph is *Problem solving* and the text further state that the students are supposed to formulate problems, to select relevant data, to pick the solution method, and to evaluate the result of their problem solving. If *problem solving* is interpreted as *solving problems* we therefore suggest that problem in that context should rather be interpreted as the same thing as a genuine problem. This interpretation is reasonable and in line with the intentions of the document according to one of the authors (G. Emanuelsson, personal communication, November 1, 2011). It is also in line with problem solving demanding creativity and time, as stated by the national curricula documents (Skolverket, 2000). The general answer to research question two is that the concept of problem can be interpreted as something similar to category 2a) something that one is ignorant of or as category 3) difficulty in this context, but the text is not unambiguous.

The concept of *problem* for Swedish upper secondary teachers

Our third research question focuses on the teachers' interpretation of the concept of problem in the course syllabus and the answer is presented in Table 1.

Interpretation of the concept of problem	**Percentage (number) of teachers**	**Comment**
1. Task to solve	50.8 % (64)	Version A
2 a. Question: something one is ignorant of	14.3 % (18)	Version B
2 b. Question: issue, matter	0 % (0)	

3 a. Difficulty: trouble	1.6 % (2)	
3 b. Difficulty: challenge	2.4 % (3)	
3 c. Difficulty: riddle	0.8 % (1)	
4. A task set in a context	6.3 % (8)	Also "a task in every day life"
5. Word tasks	7.1 % (9)	Also "word problems"
6. Circular definitions	4.8 % (4)	e.g. "a problem is a mathematical problem"
7. Other	13.5 % (17)	Mostly unclear answers

Table 1: Percentage of teachers making each interpretation of problem.

Six categories were defined in advance and several other categories were developed during the analysis. Three of them are presented as categories of their own in Table 1, and categories with no more than four answers were categorized as *Other* together with 11 unclear answers.

The most common answer was that a problem is a synonym to any task (64 teachers). This was expressed in a few different ways, but the most common answer (given by 38 teachers) was "uppgift", which is Swedish for task. Other answers categorized as *any task* were "something to be solved" and "everything is a problem." Some teachers also gave examples of what they would call problems, for example "equation with a denominator," "calculating percentages," and "a geometrical problem." Eighteen teachers used expressions similar to definition B of the word *problem*. Examples of how the teacher phrased their interpretations were "unfamiliar tasks," "when you don't know how to solve it," and "unexpected solutions." Answers of types 2b and 3 (a-c) were rare. Three teachers used the word "challenge" and one suggested that problems is what the students get when they meet difficulties in their mathematical work.

Eight teachers used expressions that were categorized as *tasks set in a context*. In this category we placed "applications," "tasks in everyday life," "real life tasks," and a few examples similar to "how to wallpaper a room." The interpretation that a problem is *a word task* was used by nine teachers. They all used the expression "text task" (Sw. textuppgift) or "reading task" (Sw. lästal or läsuppgift). Some of the teachers clarified by stating that the students should interpret something. *Circular definitions* was used by four teachers, e.g. "problems are tasks that concern problems," "problems are mathematical problems," and, "problems are problems to be solved."

Seventeen answers were put in the category: *Other*. Eleven of these were answers that were too general or unclear to classify. Some of the remaining six bear resemblances to definition B, e.g. "tasks where you have to think" or "tasks in

several steps," but are not sufficiently clear to be categorized as category 2a in Table 1. Two teachers answered; "I don't know."

The answer to research question three is that there are large variations in how Swedish upper secondary school teachers interpret the concept of problem. The dominant interpretation (more than half of the teachers), that a problem is the same as any task, is found as an alternative in several language resources, and could therefore be said to be rather sensible.

Discussion

In Swedish dictionaries, the word *problem* has several different meanings and we present a framework for analyzing statements concerning the concept. The expressions *task to solve*, *something one is ignorant of*, *issue*/*matter*, and *difficulty* are the main meanings of the concept of *problem* suggested in the structure. The formulations in the official documents concerning *problem* do not suggest a specific interpretation of the concept, and in particular, they do not exclude the possibility that a problem could be any task to be solved. The formulations concerning *problem solving*, however, suggest that the interpretations *difficulty* and *something one is ignorant of* are closer at hand. We argue therefore that the official curricula documents are not easy to interpret and that they can be seen as conveying several different meanings of the concept of problem. The interviews showed that about 51 % of the teachers interpreted *problem* in the national curricula documents as any *task to be solved*, and that the rest of the teachers interpreted the concept in many different ways. The teachers' diversified interpretations are not unreasonable considering the great variety of interpretations in the framework and the difficulty in interpreting a national curricula documents without definitions and examples.

A disadvantage with the method used in this paper is that the teachers were not asked to define or explain what *problem solving* is. Initially it was assumed by the authors that problem solving would be considered to be the same thing as solving problems, and therefore the choice of question was not problematized. However, three teachers actively suggested that problems to be solved during problem solving were of a different kind than problems in general. In combination with the results to the first research question, a natural step in following studies is to look further into teachers' interpretations of both concepts.

Is the variation in teachers' interpretation important? We argue that it is. In the national curricula documents from 1994 (Skolverket, 2000), problem solving is a very central concept, and it is implied that a significant part of the students' work in mathematics should be devoted to solving problems. Different interpretations of the concept of problem could therefore lead to very different teaching practices. One example is that Swedish students spend a large part of their time (two thirds of the lessons) during mathematics classes working with the textbook

(Bergqvist et al., 2010). For a teacher who interprets a problem as 'any task' this means that the students (already) spend two thirds of their time on problem solving. A teacher interpreting problem as something you do not know how to solve, would have to examine the textbook tasks and probably add more 'genuine problems' from other sources in order to ensure that his/her classroom practice meet the goals of the national curricula documents. Under these circumstances, the national curricula documents cannot be said to clearly guide the teachers' practice. This is in line with observations in other articles (e.g. Hill, 2001).

The difficulty with the concept of problem that we have pointed to here is a part of a more general issue concerning the use of undefined concepts in different types of curricula documents. One way to address this difficulty could be to present examples of what is meant by, e.g. problem solving, the same way as in several other frameworks (NCTM, 2000; KOM, 2003). In the Swedish national curricula documents from 1994, there are no examples at all. The combination with the absence of definitions of central concepts results in a serious situation where teachers are left virtually without guidelines on how to carry out their teaching. On the other hand, there are some indications from a parallel project that even if central concepts are clearly defined, teachers might make different interpretations. In Turkey, the national curricula documents contains a large section where problem solving is discussed and exemplified. The second sentence of the section is 'A problem should not be comprehended as a question whose solution method is already known.' Still, in interviews, teachers have different interpretations of the concept of problem (G. Karakok, personal communication, October 7, 2011).

The purpose of this study is to deepen the understanding of the connection between intentions with and interpretations of the national curricula documents in mathematics. Our main conclusions are that such documents need to be very explicit in order to give guidance to the teachers, but also that further research on teachers' interpretations and enactment of such documents is necessary.

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