

On the Use of Emphasizing Brackets when Learning Precedence Rules

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Introduction

Brackets can be used with different intentions in mathematical expressions. Here we focus on two different intentions with the symbols – *brackets as part of the precedence rules* and *brackets to emphasize precedence* (what we here call emphasizing brackets). For instance the arithmetic expression $3 + 5 \cdot 2$, according to the precedence rules (multiplication first, then addition), should yield a result of 13. If we would insert a bracket as in $(3 + 5) \cdot 2$, the expression should be calculated in order of brackets first and then multiplication, and give a result of 16. This follows the normal precedence rules that “brackets precede multiplication” and “multiplication precedes addition”. If we instead would insert brackets as in $3 + (5 \cdot 2)$, the expression should be calculated to equal 13. In this latter expression the brackets have been used to *emphasize* the precedence rules in contrast to the expression above where the brackets were part of the precedence rules. The emphasizing brackets could be considered as mathematical useless. However, it has been suggested that emphasizing brackets should be inserted for didactical reasons into arithmetic expressions (Lichevski & Livneh, 1999). In addition, it has previously been demonstrated that emphasizing brackets can enhance the structure sense in algebraic expressions (Hoch & Dreyfus, 2004) as well as in basic arithmetic expressions (Marchini & Papadopoulos, 2011).

Aim of the study

The aim of this study is to explore if the didactical intention on using emphasizing brackets could be an obstacle for students when starting to learn the precedence rules. That is, the intention is to test in which way the introduction of, didactically motivated but otherwise useless, emphasizing brackets to students has a positive effect on calculating arithmetic expressions with mixed operations.

Methodology

The data was collected in a quasi-experimental study of young students at the age of 13-14 with a test group and a control group. Both groups were given the same

pre-test (including 16 arithmetic tasks) and later post-test (including 16 other arithmetic tasks). In between, both groups were exposed to instructions on a simplified version of the precedence rules (first bracket, then multiplication, then plus and minus) including four examples of the type $(a + b) \cdot c$, and four examples of the type $a + b \cdot c$. In the test group the examples were articulated such that multiplication has higher priority and therefore can be interpreted as using extra (emphasizing) brackets around the product, though the word emphasizing were not used. In the control group brackets were not mentioned in the second type of examples. The pre- and post-tests did not contain any exercises with emphasizing brackets. In total, 169 students were investigated in this study.

Preliminary results

Computing from left to right was dominant in the students' pre-tests. The students who were exposed to the instructions using emphasizing brackets were less prone to abandon a left-to-right strategy for a precedence rule when computing $a \pm b \cdot c$ type arithmetic expressions without brackets. In detail, the increase (from pre-test to post-test) in the number of answers that can be associated to a precedence rule computation was about the same in both groups (increase by a factor of 2.2 and 2.1 in the test group and the control group, respectively). However, the number of left-to-right-related answers was decreased by a factor 2.4 in the test group compared to a factor 5.2 in the control group. Hence we cannot find didactical support in our data for the suggestion given by Lichevski and Livneh (1999) to insert brackets in order to emphasize the precedence rules in arithmetic expressions with mixed operations.

References

- Hoch, M., & Dreyfus, T. (2004). Structure sense in high school algebra: The effect of brackets. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol 30, pp. 49-56). Bergen, Norway: PME.
- Lichevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics*, 40, 173-196.
- Marchini, C. & Papadopoulos, I. (2011). Are useless brackets useful tools for teaching? In Ubuz, B. (Ed) *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3 pp. 185-192). Ankara, Turkey: PME.