Recognising Knowledge Criteria in Undergraduate Mathematics Education

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As part of a larger study on the transition between upper secondary and tertiary mathematics education, this paper reports findings from an investigation of the relation between the students' understanding of the knowledge criteria and their success during the first year in their undergraduate mathematics studies. As a methodology, we used interviews in which engineering students were given excerpts from different, more or less formal, mathematics textbooks and asked whether they can rank these texts as being "more or less mathematical". The results of our case study indeed indicate differences in the students' views that are related to their achievement.

Introduction

The analysis reported in this article is part of a larger research project (funded by the Swedish Science Foundation) about what has become called the "transition problem" from secondary to university undergraduate mathematics education. The goal of the project is to develop an integrated view of mathematical, didactical and social aspects of the transition problem. In this report we draw on one of several interviews conducted throughout their first year of university enrolment. In this interview we attempted to get insight into their awareness of the type of mathematics in the beginning undergraduate mathematics courses, in lectures and in exams (as compared to upper secondary school mathematics).

Related studies

Students enrolled in different programs, such as engineering and civil engineering, physics, other natural sciences and in teacher education have to pass mathematics courses. The outcomes of national surveys and of international research studies point to a range of issues that describe discrepancies, problems and difficulties arising from the transition from school to university mathematics learning these students face. A couple of studies suggest that discrepancies are not only a problem of specific topics to be mastered, but of a change in the type of mathematics. Mathematics at university is presented in a comparatively advanced technical language, which students perceive as more cumbersome (Hemmi, 2008). Raman (2002) infers from an analysis of textbooks that in the transition from pre-calculus to calculus students lack opportunities for coordinating informal and formal aspects of mathematical meaning, a problem addressed also by Bergsten and Jablonka (2010). Österholm's study (2008) shows the difficulties students face when confronted with a mathematical text containing symbols in comparison with a less technical version of the same text.

In a study at universities in France, Spain and Canada, the tasks to be dealt with are perceived by the students as more "abstract" (Guzmán et al., 1998, p. 749). The authors point out (pp. 752-753) that tertiary mathematics includes "unifying and generalising concepts", which set up new demands, often described as a switch from intuitive to formal mathematical thinking. In the lectures this is often related to a "Euclidean style" of presentation in the form of definition, theorem, and proof (Weber, 2004). The notion of "a fundamental conceptual divide between school and university mathematics" (Hoyles, Newman, & Noss , 2001, p. 832) has been used in this context.

In a Swedish report (HSV, 2005, p. 32), 75% of the students find the mathematics courses difficult, and 85% of the students say that the university sets up new demands. On an open question, the most common answers were "higher demands of understanding" (c. 330 in the sample of 2379, comprising 59% of the target population) and "higher level/higher demands/more content" (c. 190). The meaning of "higher demands on understanding" and "higher level" is not specified in the students' answers.

Theoretical background and research question

Our investigation attempts to clarify some of the issues related to the meaning of "higher level" of mathematics, in particular the extent to which the students are aware of changes in knowledge criteria. We draw on discursive approaches, in particular on Bernstein's theory of pedagogic discourse. For the purpose of the analysis reported here, the concepts of *classification* and *recognition rules* are of particular relevance (e.g. Bernstein, 1981). We start with the assumption that students, throughout their mathematics education, move through a range of different mathematical discourses. Differences in mathematical discourses can, for example, be described in terms of qualitative differences, e.g. foregrounding empirical or metaphorical abstract references, informal (often called 'intuitive') or formalised arguments, inductive or deductive reasoning, or in terms of the extent to which the principles of a range of mathematical activities are made explicit. When moving from high school mathematics through undergraduate mathematics to "higher levels", the knowledge becomes more strongly *classified*. The concept of *classification* describes the strength of the boundaries between discourses and groups of actors (e.g. Bernstein, 1981). Strong classification means that strong boundaries between subjects are maintained and that informal and formal knowledge are more strictly separated. The knowledge classification of undergraduate university mathematics creates specific subject-related recognition rules that differ from school mathematics and from more applied or more advanced mathematics.

In order to be successful in university mathematics activities, students need to understand the principles for distinguishing between this context, and the context of doing high school mathematics: They have to recognise the speciality of the discourse, in which they engage; they must be in possession of the *recognition rules* (ibid.). According to Bernstein, this is a necessary condition for their capacity of producing what counts as a legitimate mathematical contribution in this new context.

We investigated whether and on which grounds the undergraduate students were able to recognise weaker or stronger principles of knowledge classification, and whether there are differences in relation to their achievement.

Methodology

In order to grasp students' possession of the recognition rules, they were, in individual interviews, confronted with four different mathematical texts and asked which of those appear "more mathematical" to them. For the selection and description of the different texts, as well as for the analysis of the students' responses, we employed analytical tools developed in the context of systemic-functional linguistics (cf. Halliday & Hasan, 1989). In systemic-functional linguistics, language is modelled as interacting with the social context of its use. Each "text" (oral and written productions) is an instance of the process and a product of the social meanings in a particular context. Different aspects of meanings embedded in a text allow "predictions" of corresponding features of the context. Understanding the speciality of the context of doing undergraduate mathematics (*recognition rules*) implies recognising the speciality of the corresponding texts and vice versa.

Halliday and Hasan (1989, p. 44-45) state that a learner, while listening or reading, has to (1) "understand the processes being referred to, the participants in these processes, and the circumstances [...] associated with them [EXPERIENTIAL]", as well as the "relationship between one process and another or one participant and another, that share the same position in the text [LOGICAL]". The learner also needs to (2) "recognise the speech function, type of offer, command, statement, or question, attitudes and judgements embodied in it, and the rhetorical features that constitute it as a symbolic act [INTERPERSONAL]". Finally, he or she has to (3) "grasp the news value and topicality of the message, and the coherence between one part of the text and every other part [TEXTUAL]".

The capitalised terms in brackets refer to different aspects of meaning, which correspond to different aspects of the context, that is to (1) the field, (2) the tenor, and (3) the mode of a discourse. The field refers to the activity and topic with which the participants are engaged in which the language figures as an essential component, the tenor to the (socially significant) relationships, status and roles of the participants, and the mode to what the language is expected to achieve in the context. The employment of the framework as a methodology allows a differentiated description of knowledge classification, both in the texts as well as in the students' recognition rules.

The students

For this study, 20 first year civil engineering students from a university in Sweden were selected from five different study programmes (mechanical engineering, computer technology, physics and electric engineering, industrial economy and a programme with a focus on energy and environment, here denoted M, C, P, I, and E, respectively), so that within each programme there were students with all different combinations of low and high achievement on the diagnostic mathematics test at the beginning of their studies and the mathematics exams during the first year of study, respectively.

The interviews and the texts

The interviews on which we draw in our analysis were conducted after about half a year of the students' enrolment at university after their examinations in the introductory linear algebra and/or calculus courses. Here we only focus on one part of these individual interviews that dealt with the students' recognition of different types of mathematics. In the interviews, the students were shown four excerpts of texts (1-2 pages), all from Swedish language mathematics textbooks at undergraduate level. The texts were selected so as to correspond to different strengths in the classification of the field and variation of mode and tenor. The students were invited to compare the texts and in an open question asked whether and how they perceive them as "more or less mathematical" and to rank the texts along this dimension, if possible. The interviews were audio-recorded, transcribed and relevant parts coded according to the students' focus on field, mode and tenor of the discourse. In the following, the four texts are described in terms of these categories.

Text A (Tengstrand, 1994, pp. 52-53). Field: The text gives an example of organic growth, specified as the growth of a bacteria colony, which is described as a recursive function in two steps, and then generalized to $t = n$. By definition, $1 + p = a$, is introduced as growth factor. Substitutions into the formula are made for non-integer time periods ($t = 1/2$, $t = 1/3$). Exponential growth for the bacteria colony is then declared after deriving $N(t) = N_0a^t$. Then an example is calculated (deriving $N(t)$ from 2 values). Mode: The overall mode of the text is expository, and the steps are logically connected (*then*, *so*, *now*, *as*, etc.). The semantic choices reflect a narrative structure (bacteria growing in time), even though the tense is present tense. The coherence of the topic is achieved through repetitive use of technical terms or respective mathematical symbols, and through reference to a statement earlier in the text. Equations are printed aligned to the centre. The example at the end is framed as a procedure in symbolic notation. Tenor: An anonymous knowledgeable author speaks to an unknowing student (employing a general "we", and a "reader-we", e.g. in "as we have seen earlier"). The worked example is introduced with an imperative.

Text B (Lennerstad, 2002, pp. 238, 240-241). Field: The text deals with power functions. It introduces symbolic notations for positive exponents and basis, and then for a root function as inverse power function, justified by its suitability with "computational laws for exponents". A section on negative exponents then culminates in two theorems about growth properties of monotonous functions, and of power functions in particular. There are no proofs of these, but they appear as consequences of the exposition. The text contains a footnote about mathematical meanings of "root", which is presented as a dialogue between Hjalmar and Inge. For this "parallel text", the corresponding field is the verbalized learning activity of two students. Mode: Generally, the mode is expository, with an introduction about what is to come in the section, suggesting a didactic mode. Heading, sub-headings and the two theorems are numbered, which foreground technicality. The text also contains four graphs of functions. The description of the functions in the graphs employs non-technical terms (approaching, raising, falling, coming closer, etc.), while the rest of the text (except for the dialogue) shows a high degree of technicality. Two equations appear centred ($\sqrt[k]{x} = x^{\frac{1}{k}}$, $x^{\alpha} = \frac{1}{x^{-\alpha}}$), but also within the running text there are equations and inequalities in symbolic notation. The text is composed as a series of successive generalisations and contains grammatical metaphor. Tenor: The text employs a "we" that includes both reader and author (*we summarise*), as well as a general "we" commonly used in mathematical writing. The relationship between author and reader is constructed as one between a friendly teacher and students who try to understand. The dialogue between students in the footnote offers identification with a group of readers who try to understand the same text.

Text C (Hyltén-Cavallius, & Sandgren, 1956, p. 184). Field: The text starts with a statement and proof (also called "proof") of one form of the intermediate value theorem, which appears under the name "theorem 10" with reference to a graph of a function with several local maxima and minima. Reference to another theorem in the same book is made, and the proof in one part employs the technique of indirect proof and explicitly states assumptions. The text continues with a note that invites to conduct a "thinking experiment" in relation to whether the theorem would also be true for a function defined only for rational numbers. Mode: There is a reference to the definition of continuity in the same book. The text is expository and it foregrounds technicality as well as grammatical metaphor in all its parts except for the note. It frequently uses a general imperative (*consider*, *assume*, *let*) and a general "we" typically used in mathematics texts. Coherence is achieved through logical relation and substitution of symbols. Tenor: Throughout the text speaks an anonymous knowledgeable author to an unknowing reader in the form of an exposition.

Text D (Hellström, Morander, & Tengstrand, 1991, pp. 382-382). Field: The text has the heading "work with varying force". It starts with a general description of a process (a moving body) and poses the question that is going to be answered: *How does one calculate the work if the force K(x) changes with the distance x from the starting point?* A generalised "formula" for the problem (work as an integral) is developed through heuristic reasoning and applied to a problem of a falling body. For doing so, the formula according to which two bodies attract each other with a force proportional to their mass is stated. Throughout the text, uncommon sense interpretations of "force" and "distance", "interval" etc. are suggested by immediate use of a symbol after the words. Except for these word-symbol groups, the text employs non-specialised language, including estimation modifiers such as in "equals nearly" or "about". Statements only including symbolic notations are printed aligned to the centre. Some mathematical symbols are used in a non-technical way ($\sum$, Δx, $\rightarrow$). Mode: The overall mode is expository and in parts didactical (e.g. questions as introduction to an exposition). The text appears logically coherent, even though new themes are introduced quickly. Tenor: Throughout the text, an anonymous knowledgeable author/teacher speaks to an unknowing student (frequently employing a general "we").

According to a common characterisation of mathematics texts as focusing on technicality and grammatical metaphor, dealing with proof rather than with calculations or applied examples, as well as the expository mode and impersonal style, the ranking of the texts (from strongly to weakly classified) is CBAD.

Findings

Our general question whether recognition of the differences between the texts would be necessary for success, something "predicted" by the theory, can be answered positively. From the 20 students, only four students have ranked the texts in the order CBAD (students C6, C9, M8, P6). While one of these has moved from high (diagnostic test) to low achieving (course examination), the three others achieved high scores on both occasions. Four other well achieving students (E4, I9, M7, P3) chose CBDA, while still another (E3) offered two alternative rankings, CBAD and ABCD, and expanded on the meaning of "more mathematical". These outcomes reflect that recognition of the knowledge classification is necessary but not sufficient for success, another implication of the theory. Indeed, the three students with low scores in both diagnostic test and examination (I1, I12, M1), did not choose the ranking CBAD, but instead CBDA, DABC, CA (BD unclear), respectively. Of the remaining students, three well achieving (M5, P2, P7) ranked BC and three students who increased their achievement (E6, E8, I6) ranked CB as most mathematical, while two students with lower achievement (C7, M9) ranked A as the most mathematical text.

In the following, we present some of the arguments provided by the students. In general, they referred mostly to features reflecting the field (experiential and logical meanings) and to some extent to the mode of the discourse (textual meaning), but not so much to features that reflect the tenor (interpersonal meaning). Arguments for ranking text C as "most mathematical" and others as "less mathematical" by pointing to terminology (as reflecting the field) included:

> …because they take up more mathematical things (C6) …almost exclusively mathematical terms (C9) …easy to count the number of words that have nothing to do with math (E4) …variables, they have a curve here where the variables are declared and shown and Greek letters (I1) …strange words and only *f* of *a* not equal to *f* of *b* and all that (I9) …very arbitrary numbers *a b* and such…much palaver about bigger than zero and such stuff (M7) …one says let *y* be an arbitrary number (P3)

References to the field in favour of text C occasionally also referred to its theoretical nature and generality, such as:

> …proof for continuous functions…one defines mathematics (M7) …this gives no examples from reality (P3) …powerful mathematical proof (P6) …this is proof…with lots of intervals and continuous (E4) …these here now [C and B] deal more with the mathematics itself…describe things within the mathematics…this is then within pure mathematics…inner-mathematical (E3)

References to the mode included statements about coherence and inaccessibility:

> …theorems refer to theorems (P3) …first they say something and then they prove it…with the help of certain assumptions (M8) …strict (P6, C6) …more sectioned…with theorem and proof (E8) …a normal layman does not understand then what one talks about there (I1) …even worse (M7) …if one missed a lecture and would try to read further into it so it should somehow be such one (M8) …this is about how our teachers or lecturers go about things (M7)

A couple of students described texts C and B, and A and D, respectively, as similar and chose rankings with the first of these groups as "more mathematical", while the order within the groups varied. Many referred to the somewhat more didactical mode of the text B and to the structure, such as:

> …more explaining (C6) …explanation…as they show in the math book (C9) …also mathematical but more understandable and so because this is more text and less only expressions and symbols…more words more text more explaining [than C] (I9) …but something does that it feels less mathematical…can't actually point my finger at what this is (M7) …explains a little shows and

> explains …than just lining it up (M8) …not really a proof but they just explain what something means (P6) …written more in words actually (E4) …well more like a just flowing text (E8)

No student saw making assumptions as a feature of text B, in contrast to text C.

The texts A and D were often distinguished from the others by reference to the field. The ones who ranked D as the "least mathematical" mentioned:

> …very much examples…physics formulas (C6) …physics or what one should call that…this is kind of no well type gravitation *g* (C9) …also more a little physics about it…a relation kind of between a body and a length (M9) …not so much general…this physics…text (P7) …useful in physics (P6) …I think somehow more physics (E6) …it more applies mathematics (E3)

Reference to the field was also made by the ones who ranked text D last, such as:

> …example of a 'if we do like this so it should become like this' (C6) …some values down there…one discusses oneself forward to things that seem reasonable (P7) …not so rigorous (P6) …several different formulas…that one should transform them or do some more math than just put in a numeral (E3)

Two low achieving students ranked text A as the most mathematical, however for different reasons. One described texts B and C as equally containing "more advanced mathematics". As a general reason for the ranking ADBC, the student focussed on the mode in terms of the reader's access, for example:

> …I see as most mathematical actually to be able to get someone to understand compared with just to write formulas straight forward (C7)

The other one conveyed two conflicting views in the interview and based his final ranking on his own preferences in terms of what mathematics means and that it should be "easy to understand" (student M9):

> …yet mathematics is for me numbers and tasks…math is it is to calculate tasks to be able to apply it…it is easier to understand the mathematics when it is written with with numbers than with text…somehow more only text [in C]

However, the distinction between pure and more applied mathematics (field) was also made in the same interview:

> …they explain pure mathematics [B and C]…this one feels kind of physics [D]

As this student was one of those who improved their achievement in the examination compared to the initial test, the ambivalence could express a "transition" between different recognition rules.

However, there was a group of low achieving students, who explicitly stated that the experiential and logical meaning of the texts, that is, the field of the discourse, remained hidden to them:

> …one does not get to grasp what about what that is what one reads [C] one takes up maybe this and this [A, D] more than those two…this one I

understand actually a little more [B] this one now one can really understand [D] (C7) …this was kind of better [B] (I6) …that the brain registers more easily if it is written each in one's line [A]...that one I do not like [C]...that one I like…structured and so [D] (I12) …that one is the best [A]…simply harder to understand…here they assume things all the time…very very much theory [C] (M1) …math one should also be able to understand kind of so that there is nothing more mathematical just because it becomes more complicated (Y2)

Discussion

The outcomes of our study show the potential of a discursive approach as well as of the theoretical framework based on Bernstein's theory of pedagogic codes. In sharing the assumption of functional linguistics, in which language is modelled as interacting with the social context of its use, we assume, while talking about features of the text, what the students say reflects their experiences in the context. We observed a relation between the students' understanding of the principles for knowledge classification (*recognition rules)* and their achievement, as detailed above. While the higher achieving students were more specific when talking about the field of the discourse, for the low achieving students it seemed to be challenging to describe what the texts actually are about. These students do not seem to recognise that the principles of knowledge classification have changed in comparison with high school mathematics.

Some students took the access to the field of knowledge provided in the text as a criterion for a "more mathematical" text. The higher achieving amongst them took the esoteric nature of the texts as a criterion for ranking them as the "most mathematical". This suggests that they attribute a specific mode, namely inaccessibility, to mathematical discourse. Two low achieving students also mentioned inaccessibility, but they took it rather as the rhetorical function of the texts, that is, they referred to the tenor of the discourse. Without access to the field of the discourse, these students could naturally only focus on their status as readers. But they demanded that a mathematics text should be more accessible and one suggested that the field of the discourse should not be generalised hypothetical statements, but rather numbers and tasks. This suggestion reflects the knowledge criteria from school mathematics.

Only very few pointed to the typical "Euclidean style" of presentation in the form of definition, theorem, and proof, and pointed to making assumptions as a characteristic feature of mathematics. But the ones who described the texts that deal with examples as less mathematical, also indirectly referred to the generality of mathematics. The differences in knowledge classification between applied and pure mathematics seemed to be more obvious to most of the students, as well as the distinction between procedures and principles.

Almost all students referred to the anonymous author in plural as an active subject in their formulations. "They", perhaps the community of mathematicians or the collective of mathematics teachers, have written the texts. The students seem to assume a common tenor of the discourse. After all, despite subtle differences in tenor, all the texts speak from a position of an expert who teaches a group of similar students. The students experience themselves as participants in a community of knowers who distribute their message to the ones who do not know yet. This experience does not differentiate between high and low achievers.

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