Candy or Equation? Why do Students Get Different Explanations on the Same Problem?

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The individualisation of school mathematics teaching might be necessary in the face of the inhomogeneity of classrooms, but the choices, the students and the teacher face, might actually reinforce differences in achievement. In the following article I present and analyse data from a study about the emergence of disparity in achievement in mathematics classrooms. In the example we can see some of the effects of individualisation. The investigation shows differences in the teacher's explanations on the same problem in the interaction with different students.

Introduction

In the course of an investigation of the emergence of disparity among students in a mathematics classroom, classrooms have been video-taped for about three weeks (see Jablonka, Johansson & Rohdin 2010). The conversations analysed in this contribution, occurred in the third lesson. When students and teachers meet each other for the first time it does not take long before both the students and the teacher seem to have realised or got an idea about who apparently is good and who is not so good at mathematics. One question we ask in the research project is what kind of actions creates these images, and whether they are stable and how this affects the teaching of mathematics. In a mathematics classroom we find different domains of mathematical activity, in which the students engage. One question to ask is whether within the same classroom different students might be initiated into different domains.

We start with the observation that in mathematics classrooms an "intermediary domain" is constructed that functions as mediation between the everyday discourse of the students and the formal mathematical discourse with its specific grammatical features and technical terms:

> "As Anna Sfard shows us, in discussing the limits of mathematical discourse, the differences in the 'meta-discursive' rules between everyday discourse and mathematical discourse require us to develop a well-defined intermediary between the two." (Umland & Hersh, 2006, p.9).

Typically, this "intermediary domain" in mathematics classrooms consists of different types of word-problems. But there are other traces of the everyday domain in the form of images, didactic material, and metaphors. But the intermediary domain is not "well-defined". It becomes a shared field of activity through a process of institutionalisation in school mathematics as its "public domain" (Dowling, 2009). In our approach, we are in general interested in the potential of using intermediary domains as a base for developing mathematical concepts and methods, especially through interaction with students. This includes also possibly limiting effects of such interaction if a reproduction of everyday discourse is the outcome. Our interest focuses on the type of knowledge to which different groups of students might or might not gain access. There might be a general problem in a classroom practice that offers tasks for students without specifying the knowledge domains that form the basis for their solution (Gellert & Jablonka, 2009; Jablonka & Gellert, 2011). In the conversations analysed below, one sees that students and teachers' actually operate in different knowledge domains when they work out solutions to contextualised tasks.

Theoretical background

School mathematics, in its different versions, is related to different curriculum conceptions. These can be seen as an outcome of a process of dual recontextualisation (Jablonka & Gellert, 2010). Recontextualisation consists in subordinating one practice to the principles of another. In school mathematics, features of the practice of developing mathematical knowledge in an academic practice (such as investigation, systematisation, proof and refutation) are subordinated to pedagogic principles. But school mathematics also recontextualises domestic everyday practices, and in some curricula, also vocational practices. Consequently, word-problems are a result of a dual recontextualisation. Thus they constitute hybrids between domestic and mathematical knowledge (the "intermediary domain"). When students have to solve tasks that do not contain a specification of the knowledge domain on which a solution is to be based, different strategies allow the development of more or less generalised mathematical knowledge or more or less localised knowledge. In order to account for the emergence of such differences, Dowling's (2009) pedagogic strategies can be used as a methodological tool for interpreting interaction on the micro-level in terms of these strategies. These are the strategies reinterpreted to be used as a methodological tool.

Specialising consists in distinguishing different cases of a method or concept and developing specialised means for dealing with them, including specialised terminology. The range of application of a practice is thus reduced by specialising.

Generalising includes describing different cases in relation to a common principle and so expands the range of application of a practice.

Specialising and generalising offer a route to developing a mathematical practice of which the principles are made discursively explicit and can be shared, that is, institutionalised.

Localising is the strategy of constructing one particular local example and thus it offers no route to a principled mathematical discourse.

Articulating consists in pointing out different examples of a method, without making the underlying principles explicit.

The strategies of localising and articulating refer to (school) mathematical activities, for which the criteria are not, or cannot be made explicit. These include, for example, developing a mathematical structure that describe empirical processes and structures ("modelling"), finding a deductive proof for inductively developed principles, solving a word-problem, making an appropriate drawing to illustrate an argument, or applying a solution method to a range of different examples. In the conversation analysed in this article, the students are engaged in solving a word problem. For this activity there are no explicit criteria available to the students and there are no hints in which knowledge domain they have to solve it.

Background

The data is from a classroom with students aged fifteen to sixteen, attending the first year of Swedish "gymnasium". This is the third lesson after their very beginning at the new school. This means that the students and the teacher do not know each other from before. The students are working mostly individually with the textbook tasks and they are sitting in small groups with four students in each group. The task is from the first chapter of the textbook and deals with understanding and analysing tables and charts. The episodes have been chosen by identifying those tasks most students ask about (not necessarily in the same lesson). From these conversations about the same task two have been chosen that differ most in the use of strategies. The students are working with the task individually or occasionally in small groups. All excerpts are from the same lesson where these two students, here called Sara and Marcus, are asking the teacher for assistance with the task. The two students are not sitting close to each and independently ask the teacher.

The task

One month Fredrik got a phone bill of 226 skr. He had dialed 37 calls and sent 14 sms. How long has he been talking?

Mobile for all	Fees
Single payment	250 skr
Fee/month	59 skr
Opening fee	0,59 skr
To mobile net fee/minute	0,59 skr
To regular phone fee/minute	0,59 skr
Sms	0,75 skr
Mms	2,25 skr

This task is a word problem task and at a first glance could be placed in the public domain, but is in fact in the intermediate domain. The task appears to be authentic (in the public domain) but in fact it is actually placed in the intermediate domain because in this task you need to use just the right amount of everyday knowledge and mathematics. You need to know that the table above is not the bill, it is information about the subscription and that is everyday knowledge or as Dowling (2009) would say public domain knowledge. On the other hand you need to realise that in this specific task it does not matter what kind of calls you make because the cost is the same whether it is a call to another mobile phone or to a regular phone and that is in the intermediate domain. In the everyday the kind of calls you make, makes a difference in what costs you get. Furthermore the question about how long he has been talking is not clearly connected to the phone bill.

The Data

The following transcript is translated from Swedish. It is translated in a way that also shows the way of using the language in these conversations. Due to interruptions and other disturbances it might be hard to follow every step of the conversation but this shows clearer the use of the different strategies and the divergence in the conversation.

The first student is Sara. She is raising here hand and the teacher walks up to her.

1	Teacher:	hi is everything ok
2	Sara:	yes
3	Teacher:	mm okay
4	Sara:	on this
5	Teacher:	he gets a phone bill of two hundred twenty six crowns
6	Sara:	mm
7	Teacher:	dials thirty seven calls and sends fourteen sms and then there is a fee for how long he talks

8	Sara:	mm
9	Teacher:	what kind of strategy do you intend to use here
10	Sara:	I was thinking/
11	Teacher:	what do you want to know
12	Sara:	how long he's been talking
13	Teacher:	yes
14	Sara:	isn't it the average of every call you calculate
15	Teacher:	yahh
16	Sara:	then you can check how much one sms costs
17	Teacher:	yes
18	Sara:	that times fourteen
19	Teacher:	yes perfect
20	Sara:	take away that from that
21	Teacher:	yes and then he have dialed thirty seven calls then he has
22	Sara:	yes that's the thing i don't know then it's that times thirty seven
23	Teacher:	yes that is to and only to be allowed to start talking
24	Sara:	[inaudible]
25	Teacher:	yes and that is what is left
26	Sara:	that is how much he has been talking
27	Teacher:	you know right that is what he has called for
28	Sara:	then you take that and divide with
29	Teacher:	and then you check what every call costs eh… and then you have an amount of money and you know how much every call costs then you can calculate how many calls that was
30	Sara:	[inaudible]
31	Teacher:	mm start with sorting out and check how much money he has got left
32	Sara:	ok
33	Teacher:	so you figure out how to calculate how many
34	Sara:	ok
35	Teacher:	if it does not work i'll come back

The following conversation occurs 8 minutes later after the teacher has been talking to other students. Sara is rasing her hand again and the teacher walks up to her.

36	Sara:	I can't get it to work
37	Teacher:	was it still wrong
38	Sara:	yes
39	Teacher:	ok now you have
40	Sara:	it should be this much but I don't know how to do that
41	Teacher:	eh is this that
42	Sara:	no this was it
43	Teacher:	yes there
44	Sara:	that has nothing to with
45	Teacher:	no ok … is that what he has called for
46	Sara:	yes that is what is left when I took away
47	Teacher:	ok /sms/ if that had been two crowns per minute mm och you have had this much money mm how would you have calculated that
48	Sara:	no idea
49	Teacher:	if you had ten crowns and you by candy that costs two crowns each mm how many candies have you bought
50	Sara:	five
51	Teacher:	how did you do
52	Sara:	I took how many times two becomes [inaudible] ten
53	Teacher:	yes perfect
54	Sara:	it's ten

In line 5 we can see that the teacher is localising by repeating the task and continues to repeat the task in line 7 and hence is still localising. In line 9 the teacher is posing an open question when asking about the strategy and in lines 10-11 the teacher interrupts Sara before she answers the question and localises again by returning to the specific question in the task. In line 12 Sara responds to the teacher's question and is localising, but then in line 14 she is asking a question on how to calculate; even though she refers to the task she is using specialised language. Then Sara continues to localise in line 16 by referring to the task, in particular to the sms. In line 18 Sara is using a mathematical specialised language and is not referring to the task, in line 19 the teacher confirms Saras specialising. Sara continues without referring to the task, by means of specialised mathematics language in line 20. However, in line 21, we

can see the teacher localising again by returning to the task. In the following lines we can see the same pattern: Sara is asking for the method or answering with specialised language (except in line 26 but there it is in response to the teacher).

In line 33 the teacher is asking or stating "so you figure out how to calculate". In Swedish the teacher uses "klurar ut". The English "figure out" does perhaps not sufficiently capture that it is something difficult and not straight forward. Thereafter Sara is left alone for about ten minutes, while the teacher is talking to other students. In the following conversation a similar pattern occurs: Sara is asking for ways how to calculate and the teacher is localising. In line 47, the teacher is asking for a way of calculating with specific data. Sara's answer, "no idea", amounts to an even more localised question where the teacher (line 49) introduces prices for candy and asks how many she could buy with a given amount. In the context of the whole conversation with Sara, this is an articulating strategy through pointing to another example for the same method. Altogether, in this conversation Sara is localising and specialising by means of reference to the task, but is also asking for the principle of how to calculate. The teacher is localising within the context of the task and also in his articulation of the example with the candy.

In the following, Marcus and the teacher talk about the same task. This conversation occurs 10 minutes after the second conversation with Sara in the same lesson.

55	Teacher:	is there a problem
56	Marcus:	nnn that one…take away…and one point zero eighty crowns
57	Teacher.	What is the costs in the two hundred twenty six crowns
58	Marcus:	the sms the calls sms
59	Teacher:	sms and so the thirty seven calls and that one … right … openingfee
60	Marcus:	then those two
61	Teacher:	yes there is one more thing
62	Marcus:	fee per month
63	Teacher:	right mm now we find out how much has he spent of those two hundred twenty six with the fee opening fee per call and those sms
64	Teacher:	and those hundred fifty six and fifty
65	Marcus:	mm
66	Teacher:	that should be enough for the calls

67	Marcus:	mm ok and take away fifty nine times thirty seven too… since I know it's thirty seven calls
68	Teacher:	yes but that's the thirty seven calls he made the question is how long has he been talking together during these thirty seven calls
69	Marcus:	how do you know
70	Teacher:	have you noticed it is the same price yes per minute to mobile as to regular phone so it does not matter what kinds of calls
71	Marcus:	I know on more cost that is thirty seven times zero point fifty for the opening fee because that has to be per call
72	Teacher:	but that you took
73	Marcus:	no that was the sms
74	Teacher:	no you didn't take that it was only the sms yes perfect then
75	Marcus:	fifty nine times
76	Teacher:	mm ... ok ten fifty mm… if you have that left to call for how many calls does he have or how many minutes has he been talking if it costs fifty nine öre ya
77	Marcus:	two hundred twenty eight point
78	Teacher:	well the point we don't care about only whole minutes
79	Teacher:	mm have you reasoned that you take each cost separately and then what's left and then you took the next cost and then what was left
80	Marcus:	mm
81	Teacher:	another way
82	Marcus:	division
83	Teacher:	well yes that is also something you could do x calls and then set up everything and put it equal to the two hundred twenty six that's something one could do
84	Teacher:	and if you should be able to calculate the total cost first before without those calls fee calculate the fifty nine plus thirty seven times zero fifty nine plus fourteen times sms zero seventy five and so get a total sum and then subtract and see what is left
85	Marcus:	yes

86	Teacher:	it is possible to solve in a number of ways
87	Marcus:	mm I did the equation first but I could not solve it because I forgot that fee
88	Teacher:	you forgot the fifty nine well I did the same over there
89	Marcus:	ya

In line 55-67 Marcus and the teacher are both discussing the task and they hold on to a localising strategy as they refer to the context of the task in each utterance. Then the question about what kind of calls is posed in line 68-70. Here Marcus is using everyday knowledge by realising that there usually is a difference between the cost to a mobile phone compared to a regular phone and hence he is localising even more. In line 79 the teacher is specialising by describing Marcus' ways of calculating, then, in line 81, the teacher is generalising by asking for another method to solve the task. In line 83 the teacher is specialising by describing a solution to the task in mathematical language including variables. In line 84 the teacher introduces another method for how to solve the task and hence he is specialising.

Discussion

There are obviously different strategies in operation in the conversions with Sara and Marcus. As discussed above, only generalising and specialising offer access to a principled mathematical discourse. Thus, the different strategies used by the teacher can be seen as contributing to the disparity in achievement in this classroom. It seems that Marcus gets more chances to solve the problem as a *mathematical* problem. One question to ask is on what the difference could be based. Do the teachers' contributions and questions emerge from within the particular conversations, forced by the interactional dynamics? This would mean, the teacher tries in each case to follow up the strategies used by the students. However, a look at the students' turns only reveals that both, Sara and Marcus, use specific data from the task and do not attempt to generalise. In fact it is Sara who requests a general strategy and Marcus is using even more localising strategies.

In light of the outcomes of the analysis of the conversations presented above, the question to be followed up in the future course of this research project is to trace back all conversations these two students had previously with the teacher. As it is only the third lesson, it is important to find out how and when the differential discourse of the teacher started. Also, it is planned to relate the outcomes to the students' social and economic background and to their mathematics achievement at the end of the course.

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