Affordances and Their Mediating Artifacts as Instruments for the Collaborative Design of Innovative Mathematical Learning Activities

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The unfolding of an affordance, as an opportunity for action*, during a learning activity requires the learner to interact with mediating artifacts. The design of a learning activity involves appropriating affordances and embedding them in the activity in such a manner that the learner is invited to unfold the affordances, through interaction with their mediating artifacts in accordance with pre-defined hypothetical learning trajectories. In this paper, the notion of affordances is used explicitly in the discussion of two previous research efforts. We argue that the notion of affordances, which was tacitly used in these efforts and aligns well with the methodology of scenario-based design, may be used as an instrument for the collaborative design of innovative mathematical learning activities.*

Introduction

Researching in collaboration with others is as much an ideology as a methodology. By joining forces, drawing on each others' competencies, and working as a team, we "empower and intellectually liberate its participants" (Gershon, 2009). Such a process of intellectual liberation is often characterized by improvisation, imagination, and messiness (ibid.). While such features are generally not highly regarded in research, they may afford creative opportunities and stimulate explorations with the potential for innovation. In order to direct the innovative force in collaborative efforts and meet the scientific demands for research studies, we may draw on the coordination of expertise across disciplines. The involvement of team members with different research backgrounds enables the productive networking of theories (Prediger, Bikner-Ahsbahs, and Arzarello, 2008), specifically in the coordination of complementary research traditions and the combination of research findings from different fields.

As a researcher in mathematics education, the author has been engaged in several collaborative research efforts together with researchers in media technology, technical developers and in-service teachers. We have specifically explored the use of information and communication technologies (ICT) for the learning of mathematics, as ICT offer new possibilities for what we can do and where we can do it. In several of our projects, we have chosen to pursue the innovative design of activities with technologies for outdoor settings, inspired by the educational potential of ICT and the claim that “insofar as we are concerned with spatial ideas in mathematics as opposed to just visual ideas, we must attend to large, full-sized space, as well as to space as it is represented in models, and in drawings on paper” (Bishop, 1980, p. 260). We find it particularly useful to address the design of activities for full-sized space as they naturally support students’ enactive mode of action, which is more rarely exploited in school activities than the more abstract iconic and symbolic modes (Bruner, 1966).

The collaborative efforts of our research team involve extensive design considerations. While some team members know how to make use of the technologies, other team members can relate to opportunities for learning mathematics. The collaborative development of innovative technology-enhanced activities for the learning of mathematics calls for effective communication and establishing common ground within the research team, in order to direct the joint efforts. During the past years we have noticed a shift of attention in the team negotiations from discussing artifacts as entities towards considering their inherent qualities, that are more directly related to the provision of opportunities for learning.

In this paper, we reflect on our previous work and discuss two of our design efforts with focus on design considerations regarding *affordances*, defined as opportunities for action. Our research objective is to argue that the focus on affordances in the design process facilitates innovative and creative considerations regarding the appropriation of artifacts that mediate these affordances.

Methodological considerations regarding design of tasks and activities

Design research may be considered as a reaction against the dominating educational research tradition of “outcome evaluation”, which does not explicitly consider the provision of opportunities for learning (Glass, 1976; Cobb, Confrey, diSessa, Lehrer, and Schauble, 2003). Cobb and colleagues (2003) suggest that there is a need for explicitly addressing the issue of educational improvement by “bringing about new forms of learning in order to study them”. Two key aspects of design research are the central position of the design of teaching activities and the cyclic character that allows adjustment and improvement of the activities (Drijvers, 2003). One design cycle may be described in terms of three natural phases; the preliminary design phase, the teaching experiment phase, and the phase of retrospective analysis (ibid.). In this paper we put focus on the preliminary design phase that involves the development of a preliminary activity and a prospective analysis of this activity, with the purpose of adjusting the preliminary activity to better support the specific mathematical features expected to be explored by the students.

Design research aligns well with mathematical task design, where the notion of activity refers to the comprehensive learning environment which involves one or several mathematical tasks (Liljedahl, Chernoff, and Zazkis, 2007). While the process of mathematical task design is often described sequentially, by first considering the design of mathematical tasks and thereafter considering their pedagogical embedding in activities (ibid.), design research considers the design of the comprehensive activity, including the tasks, guided by preliminary learning objectives which are operationalized through constructions of hypothetical learning trajectories (HLT). These HLT account for students' possible interaction with the activity and aim at achieving the specified learning objectives, which may also be adjusted in the design process (Drijvers, 2003).

Mathematical tasks often involve a variety of artifacts, for example building blocks, rulers, or calculators. The choice of artifacts for an activity affects what mathematical features the students will explore while engaging in the activity (Stacey, 2011). The artifacts are chosen, or specifically designed, according to their potential for the exploration and unfolding of specific mathematical features. Designing for this process of exploration and unfolding involves considerations of pedagogical and social arrangements for the learning environment (Kirschner, Strijbos, Kreijns, and Beers, 2004).

Several aspects of our design efforts require careful considerations, primarily regarding the specification and realization of mathematical learning objectives, the functionality requirements of technical applications, and the pedagogical and social arrangements for the activities. In order to stimulate effective communication within the research team about these diverse and complex issues, we make use of the methodology of scenario-based design (Penuel, Roschelle, and Shechtman, 2007) and account for the HLT in terms of narratives. These narratives are stories that describe students' possible interaction with the learning environment in a fashion that can be followed and understood by all members of the research team. A crucial aspect of these narratives is to account for students' actions on artifacts that are (made) available in the learning environment.

According to Vygotsky's notion of explicit mediation, the students' interaction with the environment is mediated by its artifacts (Wertsch, 2007). In activity theory, the artifacts are said to mediate an *object of activity* as an object, constructed by an actor, that "singles out those properties that prove to be essential for developing social practice" (Engeström, 1999, p. 380).

In our case, the social practice concerns mathematical activity systems and the objects of activity are directed towards the solving of tasks within these systems. This idea of actor-constructed objects of activity may be useful when attempting to give an account of the actions that are actually performed by an actor during an activity. However, as the prospective analysis in design research concerns the prediction of students' future actions rather than the description and explanation of their current actions, we have to account also for *potential* objects of activity, which may be related to a variety of mediating artifacts.

Affordances mediated by artifacts

We now proceed to make a shift from “actual” to “potential” objects of activity. This shift of perspective makes it possible to consider the (potential) objects of activity independently of the artifact and instead consider its inherent qualities. Such a quality, or *affordance*, may be defined as an *opportunity for action,* as a *quality that the environment offers to an actor*, whether the actor makes use of it or not (Kirschner et al., 2004). The affordances within a particular environment “must be perceivable and meaningful so that they can be used and must support or anticipate an action” (ibid., p. 49). For example, a knife naturally affords cutting, but it also affords peeling, stabbing, and several other actions. The realization of a certain affordance is related to specific actions during an activity and depends on the actor’s capabilities, perceptions and inclinations. These perceptions and inclinations may be stimulated and encouraged through the provision of appropriate design features within the activity. For example, if a knife is exposed together with a piece of meat, it naturally affords cutting. But if the piece of meat is replaced with an apple, the knife affords both cutting and peeling. The activity may be designed to leave the decision whether to cut or peel to the actor, with or without specific stimulation. The most extreme form of stimulation is the explicit *prompt*, such as “cut the apple in half” or “peel the apple”. Explicit prompts play a particularly important role as design variables, to afford reflection and guide students’ actions in otherwise self-directed activities (Sollervall, Otero, Milrad, Johansson, and Vogel, accepted).

From a designer’s perspective, a specific affordance is naturally identified as an inherent quality of a specific artifact. However, the identification of a specific affordance allows it to possibly be appropriated to another artifact that better supports the mediation of the specific affordance. In this sense, affordances may serve as a natural instrument for the design of learning activities.

The affordances concern not only the final goal of an activity but also the intermediate experiences we would like the students to achieve while they engage in our activities. The desirable affordances appear naturally when we describe the HLT in terms of narratives. When the desirable affordances have been identified, we may engage in the appropriation of specific artifacts that serve as efficient mediators of these affordances.

On the use of affordances in educational research

The notion of affordances has been around in computer science for over 20 years (Bower, 2007). A common approach in the design of computer supported learning activities is to facilitate well-known affordances by using a computer. For example, the affordance for reading can be unfolded by scrolling down on a computer screen instead of turning pages in a printed book. This approach builds on the criticized assumption that traditional pedagogies provide sufficient guidance for the design of computer supported learning activities (Kirschner et al., 2004). Further criticism concerns the issue that too much attention has been given to the particular media used (ibid.). Regarding the design of computer supported collaborative learning (CSCL) environments, Kirschner and colleagues (ibid.) suggest to put focus on affordances for collaboration by considering the technological, educational, and social prerequisities for a collaborative activity. These three prerequisites refer to factors of the environment and describe the origin of the affordances. The affordances themselves are specified in terms of affordances for task ownership, task character, and task control (ibid.).

A similar approach based on the identification and investigation of different types of affordances, within not less than eleven delineated categories, has been explored for the purpose of matching learning tasks with learning technologies (Bower, 2007). The author suggests a design methodology which involves analyses and coordination of 1) the affordances that can be deployed on specific technological resources, and 2) the affordance requirements of the tasks with respect to educational goals.

Affordances may be classified in a variety of ways. From a pragmatic point of view, the relevance of a suggested classification depends on its power in supporting processes within a specific domain of application. In the domain of mathematics education, it is essential to consider and support affordances for *representation* of mathematical objects and ideas that are the intended focus of an activity. The representations may be in the form of pictures, diagrams, and symbols, but may also be based on physical manipulatives, including gestures where the human body is used as a manipulative (Sollervall, 2011).

The interplay between affordances for representation and communication has been exploited for educational purposes in the development of the particular dynamic mathematics software SimCalc MathWorlds® (Hegedus and Moreno-Armella, 2009). While the communicational affordances mainly concern pedagogical features, the representational affordances concern mathematical features. Hegedus and Moreno-Armella argue that the interplay between these communicational and representational affordances results in the emergence of *representational expressivity*, which enables the users to express themselves through speech acts and physical actions. This representational expressivity, as a didactical affordance for the learning of mathematics, allows for the linking of individual students' cognitive efforts to public social displays (ibid.). The design of learning situations involving SimCalc MathWorlds® is guided by strategies for scaffolding students' active multi-modal and multi-representational participation in the classroom, aiming at stimulating their intrinsic motivation. The positive impact of their use on students' performance has been established (ibid.).

While all the mentioned research efforts put focus on the use of digital and mobile technologies, we claim that the notion of affordance may be used to analyze and possibly enhance the didactical qualities of any learning activity. Specifically, we find that considerations regarding the interplay between affordances for representation and communication may be particularly useful for guiding future design efforts in mathematics education.

Although we specifically attempt to exploit the educational potential in digital and mobile technologies, we invite the use and integration of more traditional artifacts in our design efforts (Nilsson, Sollervall, and Milrad, 2009). We simply appropriate the artifact (or set of artifacts) that most efficiently mediates a specific and required affordance. For example, a rope may be preferred over a mobile device with GPS in a specific context if it more efficiently affords measuring in that context.

We will now proceed to show how the notion of affordance can serve as a design instrument for the development of innovative mathematical learning activities. Our first case concerns an activity where we at the beginning of the design process aimed at involving mobile technologies, but instead chose to involve material artifacts as they served us better as mediators of the desired affordances. The second case demonstrates how mobile technologies may provide an added value for learning, in terms of the efficient mediation of affordances that are not readily mediated by traditional artifacts.

The first case: The construction and measuring of large triangles

The first example of our collaborative efforts provides some insight into the innovative power of working with affordances (Nilsson et al., 2009). The notion of affordance was used tacitly, but not explicitly, by the research team in the design process.

The particular project started off as "Outdoor mathematics with mobile technologies", but ended up as outdoor mathematics *without* mobile technologies and instead involving material artifacts (ibid.). Initially, our idea was to explore geometric patterns in outdoor settings with the aid of mobile devices. However, discussions and negotiations in the research team led to the development of an activity based on students' own constructions. This approach led us to use material artifacts, specifically flag lines and metal hooks, that were chosen to mediate affordances for representation in terms of the students' collaborative construction of a sequence of large-scale geometrical figures, as depicted in Figure 1. The activity also involved a specifically designed wooden board, shaped as a square meter with a large hole in the middle. The hole made the square lighter and easy to hold, and thus afforded carrying, while the square itself afforded the measuring of both length, area, and right angles.

The students' self-directed collaborative efforts was afforded by a sequence of seven prompts with instructions for the construction of successive figures, where each next figure was described as an extension of the previously constructed figure. For example, in the second prompt the students were instructed to construct an 8 meter long line segment from one corner of the previous (5 m long) segment so that the two segments meet at a right angle. The seven prompts were given on separate sheets of paper, to afford students' focused interpretation and discussions about the specific requirements for each subtask. Furthermore, the activity involved three additional prompts, or task descriptions, that afforded reflection and focus on mathematical issues of area and perimeter. Specifically, the students were asked to determine area and perimeter of the small triangle, the rectangle, and the large triangle, as depicted in Figure 1.

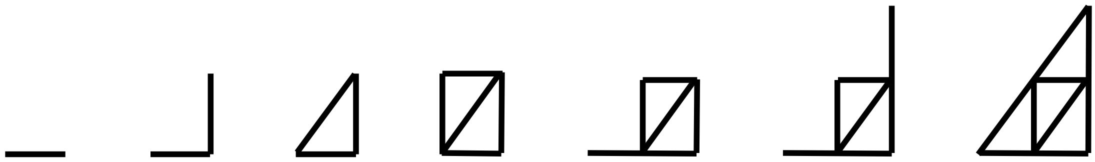

Figure 1: Intended sequence of figures to be constructed by the students.

Besides achieving the constructions and calculating perimeter and area, the mathematical learning objectives involved the conclusion that doubling lengths gives double perimeter, but also that *the area becomes four times larger*. This last conclusion may be afforded by calculations with the given lengths, or simply by the visual identification of four small triangles within the large triangle. We preferred to use the word "determine" in the task descriptions instead of "calculate", to afford the acceptance of visual inspection (and not only the use of formulae) as a strategy for determining area and perimeter of the large triangle.

The activity was initially tested in November 2008 by a group of four students in grade 6. The constructed figures were used as affordances for discussions and negotiations among the students about how to answer the specific mathematical questions about area and perimeter. The students focused on the strategy of calculation, combined with the measuring of the hypotenuse to determine the perimeters of the two triangles (Nilsson et al., 2009). In order to further stimulate the students to make use of their collaborative construction as an affordance for visual identification, the second iteration of the activity was designed to include a final written prompt to "walk into the middle of the figure and discuss what you see" (ibid.).

Besides accounting for the provision of affordances in the design process, our experiences show that it is just as important to consider the affordances that should *not* be provided. In the project mentioned above, one of our learning objectives involved the collaborative interpretation and collaborative enactment of written instructions. As a consequence, we decided not to provide any affordances for the visualization of the geometrical figures that the students were asked to construct.

The second case: Coordinating oneself with respect to given distances

The decision not to provide affordances for the visualization of drawn figures in the previous case mainly served to make the tasks more challenging and did not have a crucial impact on the learning objectives. However, in the second case that we are just about to present, the exclusion of affordances for visualization of the tasks was a crucial aspect of the design (Sollervall et al., 2011).

The idea to create the activity was initiated during a research team meeting, where a selection of technological applications were discussed for the purpose of identifying their didactical potential in terms of affordances for the learning of mathematics that could be deployed on the available technologies (Bower, 2007). The affordance that we chose to exploit for designing the activity concerned the measuring of two distances from a mobile device with respect to two fixed points, located 56 meters apart on an open field and marked with a square and a triangle, respectively. The application that we used was specifically designed by a member of the research team and involved the use of GPS technology available in the mobile device. The first of the (ten, similar) tasks within the designed learning activity is represented in Figure 2. The students started at a marked point, located 46 meters from each fixed point, and were asked to find a goal point located 42 meters from the square and 26 meters from the triangle.

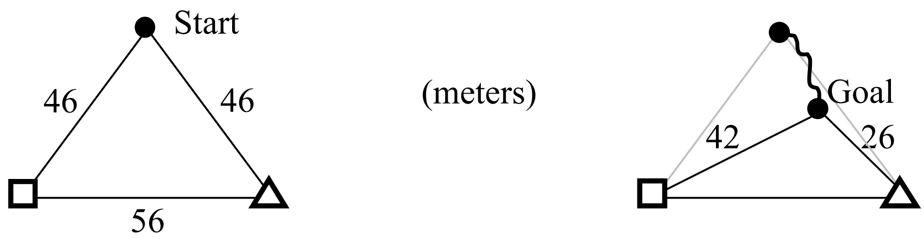

Figure 2: Visualization of the first task within the activity.

Each of the ten tasks involved in the activity may be interpreted as a Euclidean construction (Euclid's The Elements, I.22) of a triangle with three given sides:

Book I, Proposition 22.

Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

This construction may be readily accomplished on a piece of paper, by using a compass and a ruler. In that case, the actor makes use of a spatial ability which is sometimes referred to as *object manipulation*. This ability includes abilities for spatial visualization and spatial relations and concerns manipulation of spatial forms from a fixed perspective, involving an object-to-object representational system (Kozhevnikov and Hegarty, 2001). Within the psychometric research tradition, spatial visualization and spatial relations are contrasted with a third spatial ability, namely spatial *orientation*, which involves “movement of the egocentric frame of reference” (ibid., p. 745) and a self-to-object representational system. The self-to-object system activates another part of the brain than does the object-to-object system, which implies that object manipulation and spatial orientation should be considered as separate spatial abilities (ibid.).

Since object manipulation is a common activity in school mathematics, while spatial orientation is extremely rare if not non-existing, we decided to design our activity so that it would afford spatial orientation but not object manipulation, although it was certainly possible to deploy affordances for visualization on the mobile device. Specifically, the pictures in Figure 2 were not made available, as affordances for visualization, during the activity.

Concluding remarks and future efforts

The notion of affordance has been used in the TEL research tradition due to its analytical power regarding the investigation of opportunities for action that are inherently embedded in technological applications (Bower, 2007). In this paper, we have shown how affordances can be used to understand design solutions that emerge in the collaborative development of two specific learning activities. Specifically, we have argued that the design solutions regarding the provision of artifacts for these activities have been based on how efficiently the artifacts mediate the specific affordances, with particular focus on affordances for mathematical representation and communication.

As the notion of affordances have been tacitly and productively used in our design efforts we claim that they may be used as a general design instrument within mathematics education, specifically for scaffolding the collaborative design of innovative mathematical learning activities.

Working and thinking in terms of narratives, where students’ actions are described in terms of affordances, allows the entire research team to engage in creative discussions, explore a variety of opportunities, and contribute to the improvement of mathematics education by developing and testing innovative learning activities. In our future efforts, we will attempt to explicitly draw on the use of affordances for mathematical representation and communication in order to further enhance the quality of our design processes.

References

Bishop, A.J. (1980). Spatial abilities and mathematics education – a review. *Educational Studies in Mathematics, 11,* 257-269.

Bower, M. (2008). Affordance analysis – matching learning tasks with learning technologies. *Educational Media International, 45*(1), 3-15.

Bruner, J.S. (1966). *Toward a theory of instruction.* Cambridge, MA: Harvard University Press.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32* (1), 9-13.

Drijvers, P. (2003). *Learning algebra in a computer algebra environment.* Doctoral Dissertation. Utrecht: Freudenthal Institute.

Engeström, Y., Miettinen, R., Punamäki, R.-L. (Eds.) (1999). *Perspectives on Activity Theory.* New York, NY: Cambridge University Press.

Gershon, W.S. (Ed.) (2009). *The collaborative turn: Working together in qualitative research.* Rotterdam: Sense Publishers.

Glass, G.V. (1976). Primary, secondary, and meta-analysis of research. *Educational Researcher, 5,* 3-8.

Hegedus, S., & Moreno-Armella, L. (2009). Intersecting representation and communication infrastructures. *ZDM, 41*(4), 399-412.

Kirschner, P.A., Strijbos, J.-W., Kreijns, K., & Beers, P. J. (2004). Designing electronic collaborative learning environments. *Educational Technology, Research and Development, 52*(3), 47-66.

Kozhevnikov, M., & Hegarty, M. (2001). A dissociation between object manipulation spatial ability and spatial orientation ability. *Memory & Cognition, 29* (5), 745-756.

Liljedahl, P., Chernoff, E., & Zazkis, R. (2007). Interweaving mathematics and pedagogy in task design: A tale of one task. *Journal of Mathematics Teacher Education, 10*(4-6), 239-249.

Nilsson, P., Sollervall, H., & Milrad, M. (2009). Collaborative design of mathematical activities for learning in an outdoor setting. *Proceedings of CERME 6.* Lyon, France.

Penuel, W.R., Roschelle, J., & Shechtman, N. (2007). Designing formative assessment software with teachers: An analysis of the co-design process. *Research and Practice in Technology Enhanced Learning, 2*(2), 51-74.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches – first steps towards a conceptual framework. *ZDM , 40*(2), 165–178.

Sollervall, H. (2011). Modeling external representations as mediators of meaning in the mathematics classroom. *Proceedings of CERME 7.* Rzeszow, Poland.

Sollervall, H., Gil de la Iglesia, D., Milrad, M., Peng, A., Pettersson, O., Salavati, S., & Yau, J. (2011). Trade-offs between didactical and technological design requirements affecting the robustness of a mobile learning activity. *Proceedings of ICCE19.* Chiang Mai, Thailand.

Sollervall, H., Otero, N,. Milrad, M., Johansson, D., Vogel, B. (2012, accepted). Outdoor activities for the learning of mathematics: designing with mobile technologies for transitions across learning contexts. *Proceedings of WMUTE 7.* Takamatsu, Kagawa, Japan.

Stacey, K. (2011). Book review: A journey through the world of technology in mathematics education. *Educational Studies in Mathematics, 78,* 127-134.

Wertsch, J.V. (2007). Mediation. In H. Daniels, M. Cole, & J.V. Wertsch (Eds.), *The Cambridge companion to Vygotsky* (pp. 178-192). New York, NY: Cambridge University Press.