

University Mathematics Teachers' Discourses of Functions – What is Made Possible to Learn?

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My thesis project concerns the teaching of functions in undergraduate mathematics, viewed as a discursive practice. In previous studies, early reports of which have been presented at CERME 7 (Viirman 2011a) and PME 35 (Viirman 2011b), I have used commognitive theory (Sfard 2008) to describe the teaching practices of seven university mathematics teachers, from three different universities in Sweden, focusing on what characterizes the different discourses of the teachers regarding functions.

More precisely, I have used Sfards' characterization of discourses in terms of *word use*, *visual mediators*, *narratives* and *routines* (Sfard 2008, p. 133ff). The analysis of this part of the project is not yet complete, but as examples of findings I can name a classification of routines (discursive patterns), containing for instance *construction*, *substantiation* and *motivation* routines (Viirman 2011a), as well as an analysis of the way word use impacts on the clarity and accessibility of teaching (Viirman 2011b).

In this short communication however, I wish to discuss my plans for the last part of the project. Having so far given a mainly descriptive account of the discursive practices of the teachers, I now wish to investigate the possibilities of learning afforded by these practices. To this end I intend to use variation theory (Marton, Runesson & Tsui 2004), a theory focusing on conditions for learning, and also putting great emphasis on the role of language and discourse in learning, something which fits well with the discursive perspective I have used so far. I argue that the central theoretical constructs of variation theory can be made to fit into a discursive, participationist theoretical framework.

From the commognitive perspective learning is changing one's discourse, and the *object of learning* central to variation theory then becomes change in specific aspects of the discourse, for instance regarding certain discursive objects, like functions. From a discursive perspective, the *intended* object of learning is manifested in the teacher's discursive practices, while the *enacted* object of learning becomes the researcher's description of the discursive practices as they unfold in the classroom. This enacted object of learning, also called the *space of learning*, is constituted by patterns of variation, making it possible to discern the critical aspects of the object of learning. These patterns of variation can be seen as aspects of the discursive activity: "the space of learning,

which comprises different dimensions of variation, is constituted by linguistic means in the interaction between teacher and students.” (Marton, Runesson & Tsui 2004, p. 24) The similarities are obscured by the objectified language used in variation theory, but once you realize that the space of learning can be viewed as a description of a discursive activity, the compatibility is more easily seen.

What I intend to do, then, is analysing the discourses of the seven teachers, looking for the patterns of variation, examining which aspects of the function concept are made possible to discern, and which remain hidden. As for determining these critical aspects, there is a lot of research done, both by me (Viirman, Attorps & Tossavainen 2010) and a great many others (e.g. Harel & Dubinsky 1992) concerning different aspects of the learning of the function concept, and this research will be tapped into for this purpose. Here I will only briefly give two examples. One concerns arbitrariness, a characteristic feature of the modern concept of function, which among other things means that the domain and range of a function can be any type of sets. If the term ‘function’ is used synonymously with ‘real function of one real variable’, as it is by some of the teachers in my study, without making this restriction clear, then this aspect becomes unable to discern. A second example concerns linear transformations. One teacher in my study speaks of vectors $(x,y,0)$ in 3-space as two-dimensional vectors, thereby giving the impression that 2-space is always embedded in 3-space. This obscures the variation in dimension, since 3-space is not spoken of as embedded, making the transition to higher dimensions seem less natural.

References

- Harel, G., & Dubinsky, E. (Eds.). (1992). *The concept of function: Aspects of epistemology and pedagogy*. Washington, DC: Mathematical Association of America.
- Marton, F., Runesson, U., & Tsui, A. (2004). The space of learning. In F. Marton & A. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3-40). Mahwah, NJ: Lawrence Erlbaum.
- Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses, and mathematizing*. Cambridge, U.K.: Cambridge University Press.
- Viirman, O. (2011a). Discourses of functions – University mathematics teaching through a commognitive lens. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (pp. 2103-2112). Rzeszow, Poland: ERME.
- Viirman, O. (2011b). Discursive practices of two mathematics teachers on the concept of ‘Linear Transformation’. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 313-320). Ankara, Turkey: PME.
- Viirman, O., Attorps, I. & Tossavainen, T. (2010). Different views – some Swedish mathematics students’ concept images of the function concept. *Nordic Studies in Mathematics Education*, 15(4), 5-24.