

The Notion of Height – Touched Upon by Variation Theory and van Hiele Levels of Thinking

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In this pilot study two different worksheets were tested and pupils' answers were analysed. The analysis focused on what information could be obtained on pupils' notion regarding height. The two worksheets were constructed using two different theories. The worksheet based on Variation theory captured pupils understanding in relation to known misconceptions, whereas the van Hiele worksheet captured in what way pupils used formal notation.

Introduction

In Italy, Cannizzaro and Menghini (2006) constructed worksheets based upon the van Hiele theory of levels of thinking in geometry. For this study we choose to use one of these existing worksheets, and to construct a second worksheet with the same mathematical content, using Variation theory. The aim of the study is to see in what way the two worksheets assess pupils understanding of the notion of height. A subsequent question is then, if these worksheets assess differently, how can these differences be described?

Theoretical Frameworks

Two theories are used in this study, the *van Hiele levels of thinking* and *Variation theory*. For this study the first van Hiele levels are of relevance, in which the pupil goes from a visual level of thinking, through a descriptive-analytic level to a abstract- relational level of thinking (van Hiele, 2004). Variation theory describes learning as based on the concepts discernment, simultaneity and variation. The focus of attention must be drawn to the phenomenon and its critical aspects, which can be done through different patterns of variation. (Marton & Tsui, 2004)

Data

The pilot study was conducted in a lesson in two 7th grade classes (age 13) with in total 37 pupils. The data were collected during one lesson and resulted in each pupil answering one worksheet, 19 van Hiele and 18 Variation theory worksheets.

Analysis and Results

Each of the questions on the existing van Hiele worksheet was analysed and adapted according to the theoretical ideas of Variation theory. Critical aspects were made possible to discern by making use of separation, fusion and contrast. Three critical aspects concerning height were taken into account when adjusting the worksheet. A height is a *straight line, perpendicular to the base and not depending on the position*. A new critical aspect appeared in the van Hiele worksheet: pupils marked CD as a height in a rectangle ABCD with base AB.

In our setting there was no dialogue between teacher and student. Pupils did not seem to be able to discern the different aspects of the notion of height themselves and generalization did not take place, in neither one of the worksheets. The type of tasks in the van Hiele worksheet was more familiar to the pupils, which might have supported discernment but as the offered aspects on the notion of height were limited, generalization seemed more difficult.

Both worksheets addressed different aspects of the notion of height. The van Hiele worksheet showed in what way pupils were able to notate and confirmed the pupils' existing conception of height. The Variation theory worksheet showed in what way the pupils could explain their answer, and purposely challenged misconceptions. In the Variation theory worksheet it became clear that pupils show a stable notion of height, although wrong or not complete. Most likely the pupils working with the van Hiele worksheet have the same, wrong, notion but detection was not possible in that worksheet.

In the presentation we will show the choices made in the construction of the Variation theory worksheet. Furthermore the results will be exemplified.

References

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