Potential Theory and Quadrature Domains

Tomas Sjödin

Abstract: Quadrature domains, which can be thought of as domains which satisfy generalized mean-value properties for analytic, harmonic or subharmonic functions, have been under intense study for almost half a decade now (apart from their connection to some very classical potential theory which dates back a lot further).The modern theory of quadrature domains arose in connection with certain extremal problems for univalent functions, and certain minimization problems of Dirichlet integrals on Riemann surfaces. Later their connection with several other branches of mathematics, both pure and applied, have been discovered. For instance they are closely connected to the *exterior inverse problem in potential theory, Hele-Shaw flow* from fluid mechanics, *hyponormal operators* through an exponential transform, *regularity theory of free boundaries* and even *string theory*.

Most of these areas becomes quite technical, and in this talk (due to personal preference and since I am trying to reach a wider audience) I will only look at two of these connections.

First of all we will consider the exterior inverse problem, which roughly speaking is the question of whether a solid is uniquely determined by its gravitational potential outside it, and explain how this is related to quadrature domains for harmonic functions.

Then we will take a look at the Hele-Shaw flow, which arises for instance when a Newtonian fluid is being injected into a narrow gap between two parallel plates, and try to explain how this is related to quadrature domains for subharmonic functions.

I will try to mention some of my own contributions to the field, but the focus will be on a survey of the above aspects of the theory.