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Free degree of periodic self-homeomorphisms of compact surfaces

Abstract The free degree $\mathfrak{fr}(X)$ of a topological space X is the minimum positive integer n with the property that for any self-homeomorphism φ of X, at least one of the iterates $\varphi^i, i \leq n$ has a fixed point or ∞ if such minimum does not exists. It is known that for compact surfaces $\mathfrak{fr}(X)$ is finite and upper bounds for it for closed surfaces are known since Nielsen [4]; later the precise values have been contributed by Dics and Llibre [3] and Wqang [5]. In a more recent paper Wu and Zhao [6] have showed that for compact orientable surface $F_{g,b}$, of topological genus $g \geq 2$ having b boundary components, $\mathfrak{fr}^+(F_{g,b}) \leq 12g - 24$ while for non-orientable surface $N_{g,b}$, $\mathfrak{fr}(N_{g,b}) = 24g + 24$, where, the super index $+$ in the orientable case means that only orientation-preserving self-homeomorphisms are considered. Here we show that, for periodic self-homeomorphisms, these bounds can be essentially improved. We prove that $\mathfrak{fr}_{\rm per}(F_{g,b})$ is periodic with respect to b, finding in addition some period. We find explicit values of $\mathfrak{fr}^+_{\rm per}(F_{g,b})$ for $g = 2, 3$ and all b, we calculate $\mathfrak{fr}^+_{\rm per}(F_{g,b})$ for some explicitly listed values of g, b and we prove that for the remaining values of g, b, either $\mathfrak{fr}^+_{\rm per}(F_{g,b}) \leq 2(g-1)$ or $\mathfrak{fr}^+_{\rm per}(F_{g,b}) = -\chi(F_{g,b})$ and $\mathfrak{fr}^+_{\rm per}(F_{g,b'}) = \mathfrak{fr}^+_{\rm per}(F_{g,b})$ for arbitrary b' congruent to b modulo $\mathfrak{fr}^+_{\rm per}(F_{g,b})$, where $\chi(F_{g,b})$ stands for the Euler characteristic. We give similar results for $\mathfrak{fr}^\pm_{\rm per}(F_{g,b})$ which is defined by allowing also orientation-reversing self-homeomorphisms and for $\mathfrak{fr}_{\rm per}(N_{g,b})$ for non-orientable surfaces. We use mainly combinatorial approach, in study of periodic self-homeomorphisms of compact bordered surfaces, developed by Bujalance in early eighties of the last century [1, 2].

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Supported by Research Grant of Polish National Center of Sciences 2012/05/B/ST1/02171