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Algorithms for hyperbolic polygons and Fuchsian groups

Abstract Minkowski introduced an algorithm replacing generating sets of euclidean lattices, or parallelograms, by others to get a canonical form, using geometric criteria like quotients of side lengths and angles of the parallelogram. This algorithm defines a set of outcoming standard generating pairs, the well known fundamental domain of $SL_2(\mathbb{Z})$ representing the space of conformal structures on tori. J. Gilman and L. Keen applied these ideas to 2-generator groups acting on the hyperbolic plane, generated by hyperbolics, producing a final set representing hyperbolic structures of pair of pants ((0,3)-surfaces) or one-holed tori ((1,1)-surfaces). The latter case can be interpreted as transforming a triangle. But at any step of the algorithm **three** outcomes are possible: a pair to pass to the next step, a final standard pair (i. e. an acute or right angled large triangle as canonical form), or a triangle, which is too small violating Margulis- or the Collar-Lemma. This last case will bring about accidental elliptics, which show that either the generated group is not discrete, or a NEC group with unexpected signature. Unfortunately we are rarely capable of telling in beforehand, if a set of generators of a subgroup of $SL_2(\mathbb{Z})$ yields a discrete or Fuchsian group and what signature to expect. We propose algorithms to lead into this trichotomy: good group, bad group, or keep going. This we do for more general sets of generators for hyperelliptic surfaces and describe the state of the art as far as we know.