LINKÖPING INSTITUTE OF TECHNOLOGY
Mathematics Department
Numerical methods/Bo Einarsson

Instructions for the examination in

TANA39 Numerical methods for M 2004-04-24 at 08:00–13:00

General: The examination consists of seven problems.

The problems give the indicated number of points when correctly solved with **motivations and intermediate results**. Maximum number of points are 30.

For approved 12 points are required.
For the grade 4 we require 18 points.
For the grade 5 we require 24 points.

Permitted resources:
1. *Formelsamling till Numerisk Analys - en introduktion* (Eldén, Wittmeyer-Koch, Skoglund)
2. *Formelsamling till Numeriska beräkningar - analys och illustrationer med Matlab* (Eldén, Wittmeyer-Koch, Skoglund)
3. English version of any of the above.
4. TEFYMA (Ingelstam, Rönngren, Sjöberg)
5. Physics Handbook
6. At most two pocket calculators, without instruction manuals.

Duty officer: Bo Einarsson, phone 28 14 32 or 070 952 14 32. Bo will visit at around 09:00 and 11:00. Solutions will be available on the internet immediately after the conclusion of the examination: `http://www.nsc.liu.se/~boein/edu/`
The result will be announced not later than May 10.

The examinations will be shown Friday May 14 at 12–14 in the office of Bo Einarsson, House G, Room G1:181, ground floor.

Best of luck!
Bo

1. The numbers $x = 0.6 \pm 0.03$; $y = 3.76 \pm 0.04$ and $z = 2.4$ are given.

(2p) (a) Evaluate

$$f = \frac{y}{\sqrt{z + \sin x}}$$

with an error bound when z is considered exact. (An error estimate is required.)
Indicate the number of correct decimals and significant digits in the result.

(2p) (b) Assume now that we also have an error in z. How large can this error at most be in order to give f at least 1 correct decimal?

(2p) 2. (a) Calculate the sum of the following series with five correct decimals.

$$S = \sum_{n=1}^{\infty} \frac{1}{n^9}$$

(1p) (b) How many terms have to be added in the following series in order to obtain a sum with five correct decimals?

$$S = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^8}$$

(4p) 3. The function $f(x)$ assumes values as in the table below, where the x-values are exact and the f-values are correctly rounded.

x	201	202	204	205
$f(x)$	3.5	2.0	3.3	6.0

Use linear interpolation to evaluate an approximation to $f(203)$ with a complete error bound.

Sketch a figure of the function. Does it look reasonable to perform a linear interpolation here? Propose another kind of interpolation which could give a better result!

(3p) 4. A material is tested for cyclic fatigue failure whereby a stress y, in MPa, is applied to the material and the number of cycles x needed to cause failure is measured. The results are in the table below. When a log-log plot of stress versus cycles is generated, the data trend shows a linear relationship. Use the data provided in a least-squares regression analysis to compute the equation for a straight-line approximation.

x	Cycles	10	100	1000	10000	100000
y	MPa	1058	993	801	651	562

Remark: I recommend logarithms with base 10.

(5p) 5. Evaluate the integral

$$\int_1^2 \frac{\ln x}{x^{4/3}}\,dx$$

with two correct decimals. Any in your calculator integrated method for quadrature may not be used.

Compute also the additional error if the integral instead is

$$\int_1^b \frac{\ln x}{x^{4/3}}\,dx$$

where $b = 2 \pm 0.003$.

(2p) 6. (a) Determine the stability condition for Euler's method for solving the differential equation

$$y' + 4 \cdot y = \cos x$$

with the initial value $y(0) = 1$.

(4p) (b) Use Euler's method to solve the differential equation

$$y' + (4 + \sin x) \cdot y = \cos x$$

with the initial value $y(0) = 1$.

Determine approximations to $y(0.2)$ with the two step lengths $h = 0.2$ and $h = 0.05$ and perform Richardson extrapolation.

(5p) 7. Solve the boundary value problem

$$y'' - x \cdot y' - 3 \cdot x^2 \cdot y = x^3$$

Use the boundary conditions $y(0) = 1$ and $y(2) = 0$ and the step lengths $h = 1$ and $h = 0.5$ with the finite-difference method to calculate approximations to $y(x)$. Use Richardson extrapolation where possible. Sketch the solution!