LINKOPING UNIVERSITY
Department of Mathematics
Numerical analysis/Bo Einarsson/TP

Instructions for the examination

TANA39 Numerical methods for M and Mx 1998-03-14 at 8.00–13.00

General: The examination consists of seven problems.

The problems give the indicated number of points when correctly solved with **motivations and intermediate results**. Maximum points are 30.

For approved 12 points are required.
For the grade 4 we require 18 points.
For the grade 5 we require 24 points.

Permitted resources:
1. Formelsamling i Numerisk Analys (Eldén, Wittmeyer-Koch, Skoglund)
2. TEFYMA (Ingelstam, Rönngren, Sjöberg)
3. Standard Mathematical Tables *or* Physics Handbook
4. At most two pocket calculators, without instruction booklets.

Duty officer: Bo Einarsson, phone 281432

The result will be announced not later than 30 March on the bulletin board of the Dept. of Mathematics, entry B23, entry level.

Good luck!

(3p) 1. Determine all zeros of the function

$$f(x) = e^{-10x^2} + \cos(\pi x)$$

with six correct decimals.

(4p) 2. We wish to approximate the sine $\sin \pi x$ using cubic splines. We start from the table

x	0	0.5	1	1.5	2
y	0	1	0	-1	0

Use this table to determine the natural spline $s(x)$ interpolating these values.

Calculate the values of $s(0.75)$ and $s'(1.75)$.

(5p) 3. The velocity distribution of a fluid near a flat surface is given by the following table, with correctly rounded velocities v.

x mm	0	2	4	6	8
v mm/s	0.000	6.180	11.756	16.180	19.021

Newton's law for shear stress is given by

$$\tau = \mu \cdot \frac{dv}{dx}$$

where μ is the viscosity and assumed here to be 0.001 Ns/m^2. Determine the shear stress τ at $x = 0$ as accurately as possible and perform a complete error estimate.

4. The matrix A and the vector b are given as

$$A = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 0 & 1.8 & 10.1 \\ 0.2 & 4.3 & 5 & 0 \\ 0 & 0.8 & 7 & 7 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 11 \\ 42 \\ 109 \end{pmatrix}$$

(2p) a) LU decompose the matrix A with pivoting according to our rules.

(1p) b) Use the LU decomposition to solve the equation $Ax = b$.

(1p) c) Derive an upper limit for the relative error of the solution x when A is assumed exact and b may contain errors. The formula on page 12 of the formula collection may not be used for the proof.

(1p) d) Determine a numerical limit for the absolute error of the solution x above. You may use $\|A^{-1}\|_\infty = 1.6673$ and $\|\delta b\|_\infty \le 0.1$.

(4p) 5. Use Euler's method to solve the initial value problem

$$\begin{aligned} &y''(x) - 0.05y'(x) + 0.15y(x) = 0 \\ &y(0) = 1 \\ &y'(0) = 0 \end{aligned}$$

Determine approximations to $y(1)$ using the step lengths $h = 1$ and $h = 1/3$ and perform a Richardson extrapolation in order to obtain a better value.

Also calculate an estimate of the truncation error R_T.

(4p) 6. The following boundary value problem is given

$$\begin{aligned} &-2y''(x) + y(x) = e^{-0.2x} \\ &y(0) = 1 \\ &y(1) = 0 \end{aligned}$$

Solve the problem with the band matrix method, use the step length $h = 1/3$. Make a simple sketch of the approximate solution.

(5p) 7. At VTI there exists a test street with the length 1 km, measured with a puls unit and therefore its length may be considered exact. Driving a car with very precise measurement of the distance gave 1000.5 ± 0.1 m, which seems to violate the previous assumption.

Therefore assume that the car is driven not along a straight line but instead along a circle arc and determine the radius of that circle,

D = driven distance = 1000.5 ± 0.1 m
d = exact distance = 1000 m exactly
α = arc angle
R = Radius

The cosine theorem gives

$$\frac{\alpha^2}{1-\cos\alpha} = 2 \cdot \left(\frac{D}{d}\right)^2 \quad \text{(gives } \alpha\text{)}$$

$$R = \frac{D}{\alpha} \quad \text{(gives } R\text{)}$$

Determine R with error limits.