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A classification of algebraic tasks[1]

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Abstract
With a focus on content, elementary school algebra can be divided into three main subcategories – the study of equations, the study of patterns and generalisations, and the study of relations (functions). Focussing on student activity, the study of algebra involves the three phases of the algebraic cycle – translation from a "situation" to algebraic (alphanumerical) expressions, the manipulation of algebraic expressions, and the interpretation of algebraic expressions ("retranslation"). Using both these dimensions a classification of algebraic tasks into nine subcategories is easily made. A familiarity with all these kinds of tasks facilitates the use of algebra as an efficient tool for problem solving.

Preliminary empirical data indicate where student problems are located within this framework, and where special teaching efforts are needed. More general theories in mathematics education research can guide the analysis of this local problematique of school mathematics.

Introduction

Elementary school mathematics is dominated by the calculation of known magnitudes within arithmetic and geometry. When algebraic symbols (numbers denoted by letters) appear many students find mathematics difficult and, notably as a consequence, meaningless and boring. Studies on student understanding of letter symbols used in elementary school algebra show that there are big problems to view a letter as denoting a variable, as in a formula defining a function, but less difficult to use it as an unknown in a simple equation (e.g. Küchemann, 1981; Kieran, 1992). Quinlan (1992) describes five hierarchically ordered levels of understanding:

1. The letter symbol is viewed as an object without meaning
1. You need to try with only one number
1. You need to try with more than one number
1. The letter symbol is representing a class of numbers. You need to try with only one number from this class.
1. The letter symbol is representing a class of numbers. You need not try with any particular number.

How unclear conceptions like these are mirrored in student solutions to problems where algebraic symbols must be used, is exemplified in the following protocol from a test of tasks[2] given to students in grade 9 of the Swedish compulsory school.

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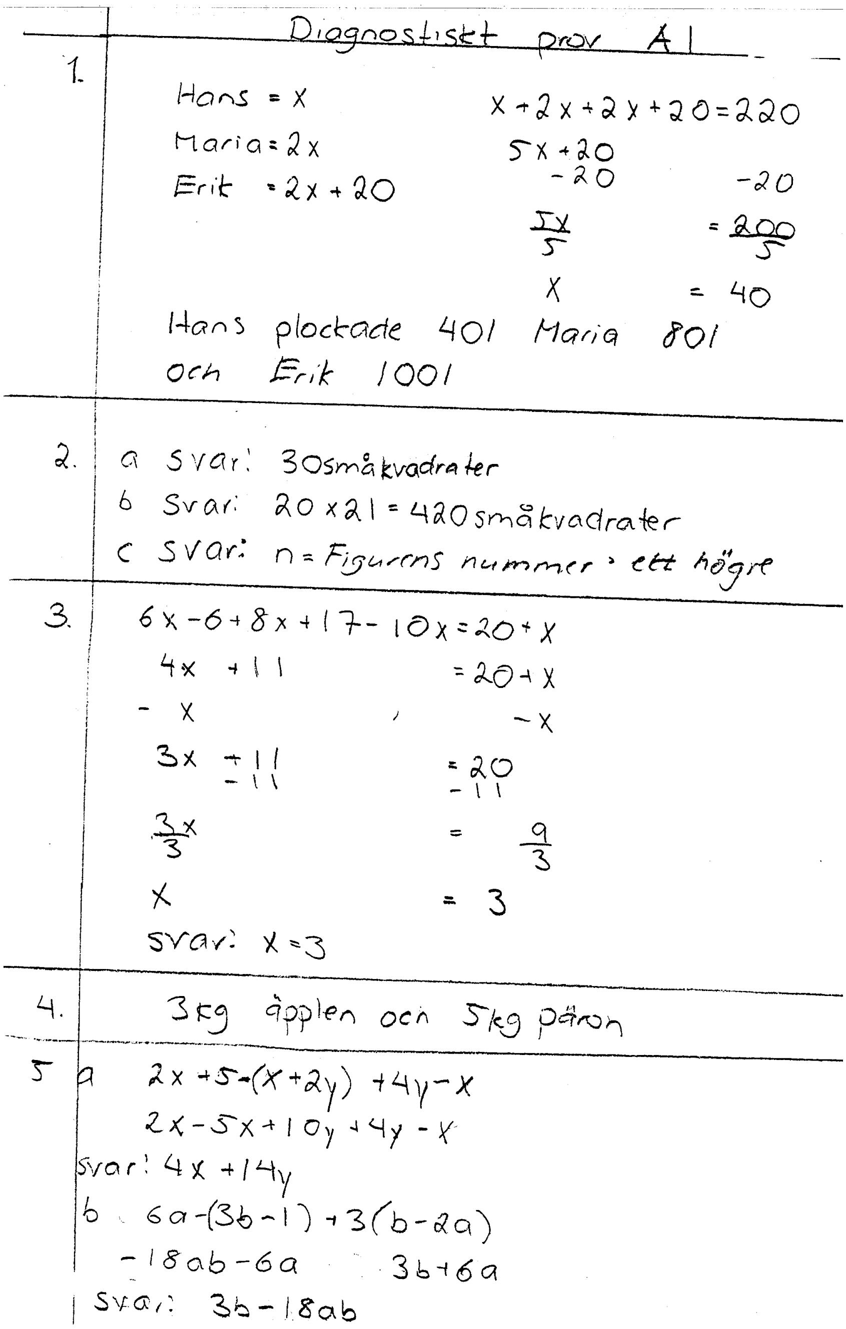
[1] Parts of this paper were presented (in Swedish) at the National Centre for Mathematics Education in Trondheim, Norway, on 18th November 2002, and published in the proceedings (in Swedish).
[2] See Appendix 1


Diagnostiskt prov A1

1. Hans = x
Maria = 2x
Erik = 2x + 20
x + 2x + 2x + 20 = 220
5x + 20 −20 −20
5x/5 = 200/5
x = 40
Hans plockade 40 l Maria 80 l och Erik 100 l

2. a Svar: 30 småkvadrater
b Svar: 20 x 21 = 420 småkvadrater
c Svar: n = Figurens nummer ' ett högre

3. 6x − 6 + 8x + 17 − 10x = 20 + x
4x + 11 = 20 + x
− x −x
3x + 11 = 20
− 11 − 11
3x/3 = 9/3
x = 3
Svar: x = 3

4. 3kg äpplen och 5kg päron

5 a 2x + 5 − (x + 2y) + 4y − x
2x − 5x + 10y + 4y − x
Svar: 4x + 14y
b 6a − (3b − 1) + 3(b − 2a)
−18ab − 6a 3b + 6a
Svar: 3b − 18ab

Figure 1. A student protocol (in Swedish) for items 1-5 of Appendix 1

Item 1 does not give this student any problem[3]. In item 2, on generalisation of a visual pattern, was solved conceptually by the student but not formally – the letter *n* was not used in the sense given in the text, and the formula given is not symbolic but rhetorical. The equation solving i item 3 is well performed (the manipulation of symbols), by a (well trained) method visible also in item 1. However, this student could not give a mathematically correct interpretation of the

[3] In the expression Hans = *x* the unit for *x* is however missing.

algebraic expression in item 4, and did not manage the simplification of the expression given in item 5. The student seems to use memorised rules of manipulation and mix them up, and does not seem to reflect upon what might be a reasonable result. Altogether, this protocol shows that this student does not have a stable algebraic knowledge – some aspects work well, others not at all. Many similar examples could be identified in different studies described below.

This paper starts up looking at a local didactical problem – to investigate what might be the reasons for the problems so frequently observed in beginning algebra, and design teaching that aims at avoiding them. Then an attempt is made to view this *problematique* from a more general point of view, where the issue of the relation of algebra to mathematics as a whole is raised. This will lead to a conceptualisation of knowledge in algebra as a discursive competence, an aspect of learning and knowing at a global didactical level. From there a road to an applied didactics of algebra, building on concepts and theories from research in mathematics education, will be opened up.

What is *x* ?

For students the word algebra is beginning to be used when letter symbols appear in their textbooks. That the same symbol can stand for different things may be very confusing to a beginner. In fact, it may contribute to an image of algebra as a big mess. The following example may illustrate this

A rectangle has a circumference of 40 cm.
a) What are the lengths of the sides if the area is 36 cm ^{2}
b) What is the maximum/minimum size of the area?

Using the letter *x* to devote the length (in cm) of the base of the rectangle, its height must be $20 - x$ cm. To answer question a) one may write the equation

$$x \cdot (20 - x) = 36 \qquad (1)$$

You find two positive solutions for *x*. So the letter *x* denotes unknown number(s) in an equation. The task is also obvious, one has to ”discover” which number is x . To find these ”hidden” numbers the following expression may be used:

$$x \cdot (20 - x) = 20x - x^2 \qquad (2)$$

This equality states a general pattern, valid for all numbers *x*. The letter *x* here stands for an arbitrary number, and the task here for the student is to explain why the equality (2) always holds.

To answer b) above on can study how the function *f* varies when *x* takes different values within the interval $0 < x < 20$, where *f* is defined by

$$f(x) = x \cdot (20 - x) \qquad (3)$$

The letter *x* is here used to describe a relation between magnitudes where *x* is the independent variable and *f*(*x*) depends on *x*.

To summarise, there are in school algebra three different uses of the letter *x* (or any other letter:[4]

[4] See e.g. Usiskin (1988).

(1) x = unknown; fix
(1) x = parameter; arbitrary
(1) x = variable; (in)dependant

Students must also be aware of the different activities (or phases) that come into in algebraic tasks. These relate to two main aspects of the algebraic language, i.e. that algebraic expressions both *carry meaning* and *can be manipulated*. This can be described by the algebraic cycle (e.g. Bell, 1996; Bergsten et al, 1997): A situation can be *translated* to an algebraic expression (e.g an equation), which may then be *manipulated* (by using algebraic rules). The resulting algebraic expression may then be *interpreted* in relation to the original situation.

This analysis points at two different dimensions of school algebra, i.e. *content* versus *activity*. Content refers to the function of the letter symbols, and activity to what you are doing to them. The above discussion shows that there are at least nine different types of tasks in elementary school algebra, as in the following figure (see figure 2):

Activity ***Content***	**Translation**	**Manipulation**	**Interpretation**
Equations	I	II	III
Patterns	IV	V	VI
Functions	VII	VIII	IX

Figure 2. Algebra as content and activity

Ses Appendix 1 for examples of school tasks for each cell of this matrix.[5] Based on figure 2 a number of research questions can be stated:

1. Which of the cells I – IX, if any, are problematic for beginning algebra students?
2. Does performance depends more on content than on activity? Or vice versa? Or is there no such structure in performance on algebraic tasks?
3. What aspects of algebraic symbolism come into play during the three different kinds of activities?
4. Are some of the cells I – IX more or less neglected in teaching?
5. What kinds of teaching/learning activities may enhance the learning within each of the nine cells, as well as the integration of them?
6. What pre-algebraic activities may prepare the student well for the letter using algebra according to diagram 2?

As an introduction to the study of some of these questions the results from three empirical pilot studies, all based on the matrix in figure 2, will be discussed here.

Some preliminary studies

To investigate the classification schema in figure 2 a test was designed (see Appendix 1), including nine items corresponding to the nine cells in figure 2. The tasks were first tried out and then used in three studies (here called A, B and C), all pilot studies for the problematique

[5] The cell numbers of diagram 2 relate to the item numbers in Appendix 1 according to the following list:
I – 1, II – 3, III – 7, IV – 2, V – 8, VI – 9, VII – 6, VIII – 5, IX – 4

described above. The results can be used only as an indicator of what is to be expected in a more systematic study, and as a basis for reflections. Therefore no detailed data are presented here, but only a general overview and a discussion. The results from study B are reported in a diploma thesis for teacher education (Nordström, 1997).

In study A the test (Appendix 1) was given for 371 students in grade 9 of the Swedish compulsory school, administrated by teacher students during their in-service training. The study included informal interviews with some of the students and teachers involved.

The same test (with two minor changes) was used in study B (Nordström, 1997), with 67 students in grade 9 of the Swedish compulsory school. A detailed analysis was performed on the student protocols. Some students were also interviewed.

In study C the same test was given to 48 students, identically at two occasions, first in grade 8 and then again in grade 9 of the Swedish compulsory school. The purpose was to study the longitudinal development of the nine content/activity categories, after one year of teaching with algebra as a substantial part of the curriculum. The tests were administered by the class teachers and the results analysed by the author.

To summarise the results, the solution frequencies were highest on the items corresponding to the cells I, II, III and VII (see figure 2), lowest for the cells V, VI and VIII, and intermediate for the cells IV and IX. Of the content areas students scored highest on equations and lowest on patterns. The activity category with the highest performance was translation (from situation to formulas) and lowest interpretation (of formulas). In the two studies also correlations and factors were studied. All these quantitative and qualitative data put together gave the following overall picture of the studied aspects:

• *Activity is more important than content area*
What the student is doing in school algebra (e.g. translating or manipulating symbols) means more for performance than what it is about (e.g. content area equations or functions). However, this conclusion is very tentative due to the influence on teaching, which was not controlled here.

• *Problems remain*
In study C the increase of performance from grade 8 to 9 was very small, which is notable with the main part of algebra teaching taking place in grade 9.

• *Variation between classes*
Apart from the normal variation present in all knowledge areas, one explanation to this can be the emphasis different teachers put on algebra in compulsory school.

• *Variation within tasks*
Without understanding or control students treat algebraic tasks in many different ways, indicating a very vague knowledge structure in algebra. (For examples, see Appendix 2)

• *Familiarity needed*
Of course training is needed for learning in all areas, but algebra seems to be especially difficult to ”discover” by your own.

• *Tasks appreciated by students*
Many of the test items looked different from the ordinary tasks in their textbooks – this was appreciated by many of the students.

• *Pre-conceived teacher views common*
Many of the teachers said the items would be too difficult for their students but were surprised when they saw the results that this was not the case. This indicates that much more can be done in algebra teaching using non-traditional less instrumentalistic tasks.

Algebra teaching

These preliminary results give rise to many reflections about the teaching of school algebra. What is to prefer, to organise educational activities with a focus on content or on activity (figure2)? Covering all three kinds of activities within one content area may be less confusing for students, keeping the meaning of the letter symbol fix. On the other side the results indicate that content is less significant than activity, and this would point in the other direction – with a focus on activity. However, there is no simple relation between theory and practice in education (e.g. Otte in Hirst & Hirst, 1988, p. 385).

Does a student learn better by training parts or by working on a more global level, in this case focussing on only one phase in the algebraic cycle or working with tasks where the cycle comes into play? Student attitude and motivation may be strongly influenced by the choice of working format. The classification by Skovsmose (2000) of different *milieus of learning* in mathematics offers an interesting frame for discussing this problem (figure 3):

	Paradigm of exercises	Landscapes of investigation
References to pure mathematics	(1)	(2)
References to a semi-reality	(3)	(4)
Real-life references	(5)	(6)

Figure 3. Milieus of learning (Skovsmose, 2000, p. 8)

Examples of (1) in figure 3 are tasks where the students shall do calculations of the type 14·15 = , 23·18 = , etc. To find, describe and explain patterns in a multiplication table is an example of (2). Type (3) is very common in textbooks of mathematics as word problems that seem to be about ”reality” but which in fact constitute an artificial reality that nobody needs to visit to solve tasks.[6] As an example of (4) Skovsmose (2000, s. 10-12) describes a classroom simulation of a horse race with bets using a plan of a racetrack and two dice. The mathematical content is about probabilities and risks. When real-life statistical data are used for training analysis of quantitative data the task is in (5) in figure 3. For describing category (6) Skovsmose (2000, s. 12-14) provides an example from a project where students went to a local farm to develop an ”input-output model” of energy in farming. According to Skovsmose a main part of

[6] An example: In stor A 0.7 kg cheese cost 35 SEK but 0.4 kg of the same kind of cheese cost 22 SEK in store B. How big was the difference in price per kilo in the two stores?

traditional teaching in mathematics concerns (1) and (3), and that using also other milieus of learning is necessary to open up the reproductive kind of school mathematics we have today, where not only (6) but also (2) and (4) are crucial milieus of learning.

Important here is of course the aim of teaching for example school algebra. Is it to be able to solve mathematical tasks of this or that kind, or is it a more general competence of developing a *symbol sense* (e.g. Arcavi, 1994; Bergsten, 1999; Piciotto and Wah, 1993) by which you have a readiness to solve such tasks but also to solve new kinds of tasks? Such a symbol sense includes for example an appreciation of the power of algebraic symbolism, to be able to read and rewrite symbolic expressions, to be able to translate between verbal/graphical representations and algebraic, to "trust" symbols and that they can lead to new knowledge (e.g. Arcavi, 1994).

When studying these kinds of problems epistemological issues are inevitable, e.g. what is algebra? The local problem of understanding the variation in student responses to algebra tasks and designing educational activities to improve the learning of algebra must be embedded in more general didactical issues, such as the functioning of the algebraic language per se, what is algebra, what is mathematics, what does it mean to know algebra/mathematics, what does it mean to learn …

What is algebra?

School mathematics is often described by its content areas, one of which being algebra. Figure 4 shows such a description, where the arrows indicate how the different content areas support each other (problems in geometry can be solved by using algebra, as an example).

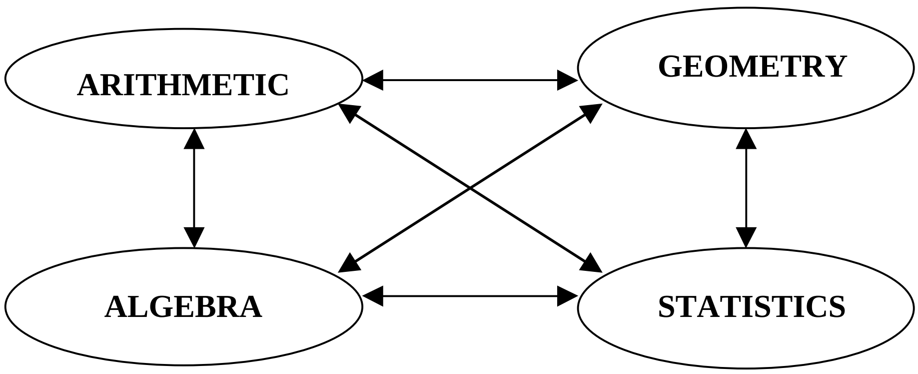


Figure 4. School mathematics as content areas

The distinction used in this paper between content and activity is in line with figure 4. However, algebra is not only about using letter symbols, it is also common to distinguish algebraic thinking from arithmetical thinking: arithmetic focuses on operations on known numbers, but with algebra it is possible to study these operations per se, i.e. to work on the structure of arithmetic and get a deeper understanding of how arithmetic works (see e.g. Sierpinska, 1995). This change of focus from operations to structures can also be observed in the history of algebra (Sfard, 1995). Bolea et al (1999, p. 141) move a step further by describing algebra not as a content area but by an *algebraization process* of other mathematical content areas:

> Elementary algebra does not appear as a self-contained mathematical work comparable to other works studied in academic core courses (such as arithmetic, geometry, statistics, etc.), but rather as a modelling tool to be (potentially) used in all mathematical curricular works and which appears to be more or less used in them.

The algebraization of a mathematical work provides, according to Bolea et al (1999), a *didactical tool* facilitating the study of the whole content area.

This way of looking at algebra, combined with more general terms for the study areas of mathematics, gives another picture of mathematics and the role played by algebra (figure 5).

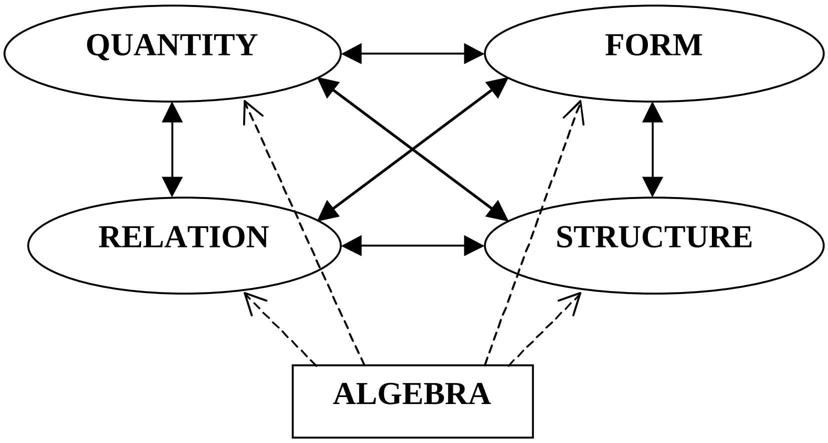


Figure 5. A picture of mathematics and the role of algebra

In figure 5 mathematics is viewed as systematic studies of quantities, forma, relations, and structures, where algebra is one (of many) tools used in these studies. You study *quantities* in arithmetic and statistics and use quantities in studies of *relations* (functions) and *forms* (e.g. geometry). Many objects in for example arithmetic and functions have common basic *structures* described in abstract algebra. Algebra is often used as a tool in all these areas. The algebraic language carries meaning and can be manipulated, which means that algebra can be described as a “double” tool:

- A *problem solving tool* – by using algebra you are better equipped to solve problems in mathematics
- A *didactical tool*– by using algebra you can get a deeper understanding of the content areas of mathematics

These remarks may shed new light on the issue of the teaching and learning of algebra.

How can you learn algebra?

Looking at algebra as a “double” tool fraught with meaning, an obvious goal for the teaching of school algebra is to develop a familiarity with this tool. Such a familiarity could be described by the term symbol sense, and can be achieved only by repeated use in situations where you feel that it is a help to solve problems and gain understanding. In the case of school algebra this means to use the different activities in the algebraic cycle (translation, manipulation, interpretation) in the different content areas (equations, patterns, functions). This may take place within different milieus of learning (figure 3), where variation is important not only for the learning per se but also for making the function of mathematics visible, and develop attitudes to mathematics that have the potential of increasing the desire to learn. The simple scheme in figure 6 can illustrate how a systematic work within a learning loop may contribute to such an increased familiarity.

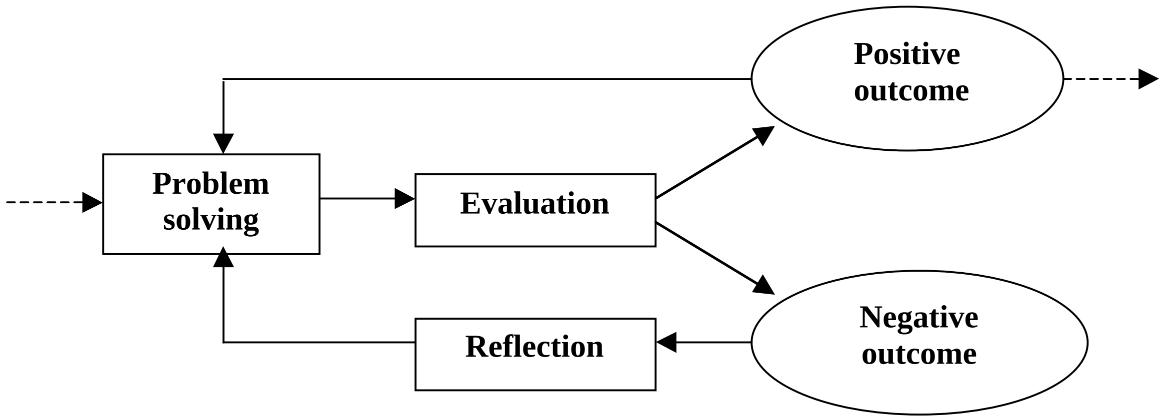


Figure 6. A learning loop for mathematics

In figure 6 the left dashed arrow indicates the experiences and knowledge you bring to a new problem solving situation, and the right dashed arrow indicates the experiences and knowledge you bring from the problem solving sequence. The work with tasks in the left box can be varied during turns in the loop to build, in the learner, richer *concept images* (Tall and Vinner, 1981).

One way of looking at figures 5 and 6 is that is all about developing an ability to communicate in mathematical terms with one self and with others, i.e. developing a discursive competence. A similar kind of *communicational competence* is described in the Danish KOM–project, where knowledge of mathematics is portrayed as a set of subject matter related competencies, classified in two categories: kinds of questions and answers; kinds of tools and languages (Niss and Højgaard Jensen, 2002). Also other subcategories like *representational competence*, *symbolic competence*, and *reasoning competence* all put an emphasis on mathematical knowledge as a discursive competence. This framework is in line with the view of algebra as a "double" tool fraught with meaning, as described above.

Such a focus on discourse has consequences on views of learning and teaching and the roles of the student and the teacher. For the student it is a communications game, to grow into a new discourse, individually and in a group, with an expert (the teacher).[7] In this game also socio-mathematical norms for what is accepted or not are established within the group (Yackel och Cobb, 1996).

Taking part in such a communicational game, where for example algebraic symbols are introduced, there is an inherent circularity: You must take part in the communication before you know how to communicate, i.e. you must use words and expressions and handle the objects before you know what they mean, how to use them or understand them (Sfard, 2001). This questions the common view that content should precede form in mathematics learning and teaching (e.g. working with numbers as ideas before introducing numerals to represent numbers). The alternative in a communicational game, as described above, is that form and content appear

[7] In literature terms like "apprenticeship learning" or "expert modelling" are used for this (e.g. Collins et al, 1989).

side by side for the student, and are treated in a discursive game towards increased understanding and discursive competence. It is a didactical challenge to handle such a game, keeping interest and motivation among the students. Some examples of didactical tools to be used in this process are the following:

- Use of different forms of representation (verbal, visual, numeric, symbolic)
- Use of mathematically relevant metaphors (Lakoff och Núnes, 2000)
- Variation of milieus of learning (figure 3)
- Development of richer concept images (Vinner and Tall, 1981)
- Awareness of the distinction between *perceptual* and *proceptual* knowledge, and the special problems involved with the latter (Tall et al, 2000)
- Systematic work with the didactical phases *Activity* → *Formulation* → *Validation* in landscapes of investigation (see Brousseau, 1997; Skovsmose, 2000)
- Design of learning loops (figure 6) that capture essential aspects of the algebraic cycle, as well as an experience of both the form and function of mathematics

The double role of algebra as providing a problem solving *and* didactical tool for the study of mathematical content areas puts algebra as one of the keys that may help to open up an integrated view and understanding of mathematics as a whole. Thus the local didactical problem set up at the beginning of this paper has found tools in more global theories of learning and teaching to pave the way for an applied *didactique* of algebra.

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APPENDIX 1 – Test items used in studies A, B and C

1. Maria, Hans and Erik had a summer job of collecting strawberries. One day they collected altogether 220 litres. Maria collected twice as much as Hans did. Erik collected 20 litres more than Maria. Write an equation by which you can find out how many litres Hans collected.

2. Below you find the first four figures in a series. You can continue to build bigger and bigger figures by the same pattern.

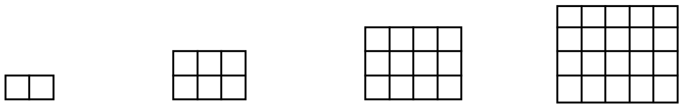


Figure # 1 2 3 4

   a) How many small squares are there in figure number 5?
   b) How many small squares are there in figure number 20?
   c) How many small squares are there in figure number *n*? (Here *n* can be any number)

3. Solve the equation $6x-6+8x+17-10x=20+x$

4. The price for one kg of apples is *a* SEK and the price for one kg of pears is *b* SEK
   What then can the expression $3a+5b$ mean ?

5. Simplify the expressions
   a) $2x+5-(x+2y)+4y-x$
   b) $6a-(3b-1)+3(b-2a)$

6. The twins Eva and Peter also pick strawberries in their summer holidays, but they work on different farms. Peter has a daily salary of 60 SEK and additionally 2 SEK for each litre of strawberries he collects. Eva has no daily salary but gets 4 SEK for each litre she collects.
   a) Write an expression that shows how much money Eva makes one day if she collects *x* litres of strawberries.
   b) Write an expression that shows how much money Peter makes one day if he collects *y* litres of strawberries.

7. One side of a (rectangular) swimming pool is fifteen meters longer than the other side. The following equation is about the swimming pool.

   $2x+2(x+15)=150$

   Write in your own words what the equation describes.

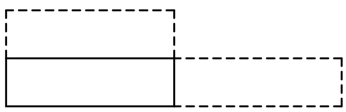
8. Anders, Jenny and Per made bigger and bigger figures of small squares by the same pattern and then wrote down an expression for the number of small squares in figure number *n*.

   Anders got the expression $1+2\cdot(n-1)$
   Jenny got the expression $n+(n-1)$
   Per got the expression $2\cdot n-1$

   Can the three expressions describe the number of squares in the same figure? (Explain)

9. Which one of the following formulas goes best to the figure below? Also explain your choice.

   $2a+b=a+2b$ $a\cdot a\cdot b=a\cdot b\cdot b$ $a+ab=ab+b$ $2a\cdot b=a\cdot 2b$



APPENDIX 2

Examples of variation within tasks from student protocols to Appendix 1

Item 1

- $x + 2x + 2x + 20 = 220$
- $x + 2x + x + 20$
- $x + x^2 + x^2 + 20 = 220$
- $x \cdot 20 + x^2 + x = 220$

Item 4

- 3 kg of apples and 5 kg of pears
- The price for 3 kg of apples and 5 kg of pears together
- You have bought 3 kg of apples and 5 kg of pears
- The apples cost 3 SEK and the pears 5 SEK
- $3a + 5b = 8c$
- Fruit salad

Item 5a)

$2x + 5 - (x + 2y) + 4y - x$

$2x + 5x - 10y + 4y - x$

$6x - 6y$

$2x + 5 - (x + 2y) + 4y - x$

$2x + 5 - 5x + 10y + 4y - x$

$-2x + 5 + 14y$

Item 6a)

- E = $x \cdot 4$
- Eva = $4x$
- T = $x \cdot 4$
- $x = x \cdot 4$
- $x + 4$ kr
- $x = 4$ kr
- $x \cdot 4$
- $x = 4y$
- $x = 4$
- $4x$
- Salary = $4 \cdot x$
- SEK = $x \cdot 4$