Linköping Studies in Science and Technology
Thesis No. 1311

The Use of Landweber Algorithm in Image Reconstruction

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Akademisk avhandling

Som för avläggande av filosofie licenciatexamen vid Tekniska Högskolan i Linköping kommer att offentligt försvaras i kompakta rummet, hus B23, Linköpings universitet, tisdagen den 12 juni 2007, kl. 10.15. Opponent är Dr. George Baravdish, Linköpings universitet, Campus Norrköping.

Abstract

Ill-posed sets of linear equations typically arise when discretizing certain types of integral transforms. A well known example is image reconstruction, which can be modelled using the Radon transform. After expanding the solution into a finite series of basis functions a large, sparse and ill-conditioned linear system arises. We consider the solution of such systems. In particular we study a new class of iteration methods named DROP (for Diagonal Relaxed Orthogonal Projections) constructed for solving both linear equations and linear inequalities. This class can also be viewed, when applied to linear equations, as a generalized Landweber iteration. The method is compared with other iteration methods using test data from a medical application and from electron microscopy. Our theoretical analysis include convergence proofs of the fully-simultaneous DROP algorithm for linear equations without consistency assumptions, and of block-iterative algorithms both for linear equations and linear inequalities, for the consistent case.

When applying an iterative solver to an ill-posed set of linear equations the error typically initially decreases but after some iterations (depending on the amount of noise in the data, and the degree of ill-posedness) it starts to increase. This phenomena is called semi-convergence. It is therefore vital to find good stopping rules for the iteration.

We describe a class of stopping rules for Landweber type iterations for solving linear inverse problems. The class includes, e.g., the well known discrepancy principle, and also the monotone error rule. We also unify the error analysis of these two methods. The stopping rules depend critically on a certain parameter whose value needs to be specified. A training procedure is therefore introduced for securing robustness. The advantages of using trained rules are demonstrated on examples taken from image reconstruction from projections.

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Linköping 2007